Hedging, Liquidity, and the Multinational Firm under Exchange Rate Uncertainty *

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This paper examines the impact of liquidity risk on the behavior of the risk-averse multinational firm (MNF) under exchange rate uncertainty in a two-period dynamic setting. The MNF has operations domiciled in the home country and in a foreign country, each of which produces a single homogeneous good to be sold in the home and foreign markets. To hedge the exchange rate risk, the MNF trades currency futures contracts that are marked to market at the end of the first period. Liquidity risk is introduced to the MNF by a liquidity constraint that obliges the MNF to prematurely liquidate its futures position whenever the interim loss incurred from this position exceeds a prespecified threshold level. We show that the liquidity constrained MNF optimally opts for an under-hedge, sells less (more) and produces more (less) in the foreign (home) country. When the set of hedging instruments made available to the liquidity constrained MNF is expanded to include nearby currency futures and option contracts, both of which mature at the end of the first period, we show that the MNF optimally opts for a long nearby futures position, a short distant futures position, and a long option position. This paper thus offers a rationale for the hedging role of futures spreads and currency options for liquidity constrained MNFs.

JEL classification: D81; F23; F31

Keywords: Currency hedging; Liquidity constraints; Multinationals

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Hedging, liquidity, and the multinational firm under exchange rate uncertainty

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Abstract

This paper examines the impact of liquidity risk on the behavior of the risk-averse multinational firm (MNF) under exchange rate uncertainty in a two-period dynamic setting. The MNF has operations domiciled in the home country and in a foreign country, each of which produces a single homogeneous good to be sold in the home and foreign markets. To hedge the exchange rate risk, the MNF trades currency futures contracts that are marked to market at the end of the first period. Liquidity risk is introduced to the MNF by a liquidity constraint that obliges the MNF to prematurely liquidate its futures position whenever the interim loss incurred from this position exceeds a prespecified threshold level. We show that the liquidity constrained MNF optimally opts for an under-hedge, sells less (more) and produces more (less) in the foreign (home) country. When the set of hedging instruments made available to the liquidity constrained MNF is expanded to include nearby currency futures and option contracts, both of which mature at the end of the first period, we show that the MNF optimally opts for a long nearby futures position, a short distant futures position, and a long option position. This paper thus offers a rationale for the hedging role of futures spreads and currency options for liquidity constrained MNFs.

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1. Introduction

In 1993, MG Refining and Marketing, Inc. (MGRM), the U.S. subsidiary of the fourteenth largest industrial firm in Germany, Metallgesellschaft A.G., offered its customers fixed prices on oil and refined oil products up to 10 years into the future as a marketing

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device. To hedge its exposure to fluctuations in oil prices, MGRM took on large positions in energy derivatives, primarily in oil futures. When oil prices plummeted in 1993, the margin calls on MGRM's futures positions were substantial. MGRM was unable to meet its variation margin payments due to the denial of credit from its banks.[1] To keep Metallgesellschaft A.G. from going bankrupt, MGRM's liquidity problems resulted in a $2.4 billion rescue package coupled with a premature liquidation of its futures positions *en masse* (Culp and Miller, 1995).

The case of Metallgesellschaft A.G. vividly demonstrates the importance of taking liquidity risk into account when devising risk management strategies.[2] Liquidity risk can be classified as either asset liquidity risk or funding liquidity risk (Jorion, 2001). Asset liquidity risk refers to the risk that the liquidation value of the assets differs significantly from the prevailing mark-to-market value. Funding liquidity risk, on the other hand, refers to the risk that payment obligations cannot be met due to inability to raise new funds. Ignorance of liquidity risk is likely to result in fatal consequences for even technically solvent firms. Prominent examples of this sort also include the debacle of Long-Term Capital Management.[3]

The purpose of this paper is to study the impact of liquidity risk on the behavior of the risk-averse multinational firm (MNF) under exchange rate uncertainty (see, e.g., Adam-Müller, 1997; Broll, 1992; Broll and Wong, 1999, 2006; Broll, Wong, and Zilcha, 1999; Wong, 2006). To this end, the MNF is restricted to use currency futures contracts as the sole hedging instrument. Liquidity risk arises from the marking-to-market process of the currency futures contracts and from the inability of the MNF to raise external funds to finance huge interim losses incurred from its futures position.[4]

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[1] Culp and Hanke (1994) report that "four major European banks called in their outstanding loans to MGRM when its problems became public in December 1993. Those loans, which the banks had previously rolled-over each month, denied MGRM much needed cash to finance its variation margin payments and exacerbated its liquidity problems."

[2] According to the Committee on Payment and Settlement Systems (1998), liquidity risk is one of the risks that users of derivatives and other financial contracts must consider.

[3] See Jorion (2001) for details of the case of Long-Term Capital Management.

[4] One might argue that the interim losses incurred from the futures position are coupled with a concomitant increase in the value of the spot position. The MNF at worst faces only a temporary liquidity problem and thus should be able to persuade its creditors to extend financing for the resulting deficit. However, it is

We develop a two-period variant model of the MNF under exchange rate uncertainty along the line of Broll and Zilcha (1992), Lien and Wong (2005), and Meng and Wong (2007). The MNF has operations domiciled in the home country and in a foreign country. Each of these two operations produces a single homogeneous good to be sold in the home and foreign markets. The currency futures contracts are marked to market in that they require interim cash settlement of gains and losses at the end of the first period. These ex-post funding needs due to the marking-to-market process create liquidity risk vis-à-vis the exchange rate risk. Following Lien (2003) and Wong (2004a, 2005), we impose a liquidity constraint on the MNF in that the MNF is obliged to prematurely liquidate its futures position whenever the interim loss incurred from this position exceeds a prespecified threshold level. The liquidity constraint as such truncates the MNF's payoff profile, which plays a pivotal role in shaping the MNF's optimal production and hedging decisions.

When the liquidity constraint is absent, the celebrated separation and full-hedging theorems in the literature on the MNF under exchange rate uncertainty apply (see, e.g., Adam-Müller, 1997; Broll, 1992; Broll and Wong, 1999, 2006; Broll, Wong, and Zilcha, 1999; Wong, 2006). The separation theorem states that the MNF's production and sales decisions depend neither on its risk attitude nor on the underlying exchange rate uncertainty. The full-hedging theorem states that the MNF should completely eliminate its exchange rate risk exposure by adopting a full-hedge via the unbiased currency futures contracts.

In the presence of the liquidity constraint, we show that the MNF optimally opts for an under-hedge should it be prudent in the sense of Kimball (1990, 1993).[5] Under-hedging is called for to strike a balance between the extent of the exchange rate risk and that of the liquidity risk. This is consistent with the normal practice of partial hedging that most companies do not use financial derivatives to completely hedge their risk exposures (Tufano, 1996; Bodnar, Hayt, and Marston, 1998).

hard to believe that creditors could understand what the MNF is doing in general, and distinguish between a hedge position and a speculative one in particular. Concerns about moral hazard problems are thus likely to stop creditors from providing funds at a time when the MNF needs them most, as is evident from the Metallgesellschaft case.

[5]Loosely speaking, prudence refers to the propensity to prepare and forearm oneself under uncertainty, in contrast to risk aversion that is how much one dislikes uncertainty and would turn away from it if one could.

Since the MNF is obliged to prematurely liquidate its futures position whenever the net loss incurred from this position exceed the prespecified threshold, the MNF's sales in the foreign market are embedded with residual exchange rate risk that cannot be hedged via the currency futures contracts. We show that the MNF demands a positive risk premium on its foreign sales. This creates a wedge between the marginal revenues in the home and foreign markets. In response to the imposition of the liquidity constraint, the MNF optimally sells less (more) and produces more (less) in the foreign (home) country, in accord with the findings of Broll and Zilcha (1992), Lien and Wong (2005), and Meng and Wong (2007). These adjustments in sales and outputs result in a lower expected global domestic currency profit accrued to and a lower expected utility level attainable by the MNF, as compared to those in the absence of the liquidity constraint.

We further expand the set of hedging instruments made available to the MNF to include nearby currency futures and option contracts, both of which mature at the end of the first period. We show that the prudent MNF optimally opts for a long nearby futures position and a short distant futures position when the liquidity constraint is present. Such a trading strategy constitutes a futures spread that involves a long or short position in one futures contract and an opposite position in another.[6] We show further that the liquidity constrained MNF optimally uses the one-period currency option contracts for hedging purposes in general, and opts for a long option position if its preferences exhibit prudence in particular. Since the liquidity constraint truncates the MNF's payoff profile, the MNF finds the long option position particularly suitable for its hedging need. These findings are consistent with the prevalent use of futures spreads and options by non-financial firms (Bodnar, Hayt, and Marston, 1998).

The conventional rationale for the use of futures spreads is a speculative one: When the difference between the prices of the two futures contracts, i.e., spread basis, is beyond a perceived "normal" level, traders employ spread strategies to take advantage of the expected price realignment (Chance, 2004). Schrock (1971) shows that futures spreads are important

[6]If both futures contracts are on the same commodity but with different expirations, such a trading strategy is referred to as an intramarket spread, a calendar spread, or a time spread. An intermarket spread is the one in which the two futures contracts are on different commodities.

because they expand the feasible set of investment opportunities beyond that defined by outright long and short positions (see also Poitras, 1989). Peterson (1977) points out that margin requirements are lower for futures spreads than for outright long or short positions. This favorable margin treatment thus offers an economic rationale for trading futures spreads. Lioui and Eldor (1998) show that futures spreads can span the opportunity set of the primitive assets. This spanning property improves the hedging effectiveness for investors endowed with non-traded cash positions in general and allows them to attain their first-best welfare levels in particular. This paper adds to this sparse literature on the optimal use of futures spreads in that the prevalence of liquidity constraints gives rise to liquidity risk because distant futures positions on which the interim losses incurred reach an alarming level have to be prematurely liquidated. Opposite nearby futures positions are called for to limit the degree of liquidity risk, thereby rendering a pure hedging demand for futures spreads (see also Wong, 2004b).

This paper is also related to the burgeoning literature on the hedging role of options. Adam-Müller and Wong (2006), Moschini and Lapan (1992), and Wong (2003a) show that export flexibility leads to an ex-ante profit function that is convex in prices. This induced convexity makes options a useful hedging instrument. Brown and Toft (2002), Lien and Wong (2004), Moschini and Lapan (1995), Sakong, Hayes, and Hallam (1993), and Wong (2003b) show that firms facing both hedgeable and non-hedgeable risks would optimally use options for hedging purposes. The hedging demand for options in this case arises from the fact that the two sources of uncertainty interact in a multiplicative manner, which affects the curvature of profit functions. Lence, Sakong, and Hayes (1994) show that forward-looking firms use options for dynamic hedging purposes because they care about the effects of future output prices on profits from future production cycles. Frechette (2001) demonstrates the value of options in a hedge portfolio when there are transaction costs, even though markets themselves may be unbiased. Futures and options are shown to be highly substitutable and the optimal mix of them is rarely one-sided. Broll, Chow, and Wong (2001) document the existence of a non-linear component in the spot-futures exchange rate relationship in five out of six currencies of developed countries. Since the underlying uncertainty is non-linear in nature, options are needed to achieve better hedging performance. Lien and Wong (2002) justify the hedging role of options with multiple delivery specifications in futures markets. The presence of delivery risk creates a truncation of the price distribution, thereby calling for the use of options as a hedging instrument. Chang and Wong (2003) theoretically derive and empirically document the merits of using currency options for cross-hedging purposes, which are due to a triangular parity condition among related spot exchange rates. This paper offers another rationale for the hedging role of options when liquidity risk prevails (see also Meng and Wong, 2007; Wong and Xu, 2006).

The rest of this paper is organized as follows. Section 2 delineates the dynamic model of the MNF facing both exchange rate uncertainty and liquidity risk. Section 3 examines how the presence of the liquidity constraint affects the MNF's optimal hedging and sales decisions. Section 4 introduces nearby and distant currency futures contracts to the MNF and establishes the optimality of using futures spreads. Section 5 introduces currency option contracts to the MNF and shows the hedging role of options. The final section concludes.

2. The model

Consider a dynamic variant model of the multinational firm (MNF) under exchange rate uncertainty *à la* Broll and Zilcha (1992), Lien and Wong (2005), and Meng and Wong (2007). There are two periods with three dates, indexed by $t = 0$, 1, and 2. To begin, the MNF has operations domiciled in the home country and in a foreign country. Interest rates in both periods are known at $t = 0$ with certainty. To simplify notation, we henceforth suppress the interest factors by compounding all cash flows to their future values at $t = 2$.

The MNF's home operation produces a single homogeneous good, x, according to a cost function, $c_x(x)$, denominated in the domestic currency. We assume that $c_x(x)$ is strictly increasing and convex to reflect the fact that the MNF's production technology exhibits decreasing returns to scale. At $t = 2$, the MNF sells x_h and x_f units of the good, x, in the home and foreign countries, respectively. The sales in the home market generate a revenue function, $r_x(x_h)$, denominated in the domestic currency, whereas the sales in the foreign market generate a revenue function, $R_x(x_f)$, denominated in the foreign currency. We assume that $r_x(x_h)$ and $R_x(x_f)$ are strictly increasing and concave to capture the idea that the MNF is likely to enjoy some monopoly power in the home and foreign markets.

The MNF's foreign operation produces another single homogeneous good, y, according to a cost function, $c_y(y)$, denominated in the foreign currency, where $c_y(y)$ is strictly increasing and convex. We assume that the two homogeneous goods, x and y, are independent of each other.[7] At $t = 2$, the MNF sells y_h and y_f units of the good, y, in the home and foreign countries, respectively. The sales in the home market generate a revenue function, $r_y(y_h)$, denominated in the domestic currency, whereas the sales in the foreign market generate a revenue function, $R_y(y_f)$, denominated in the foreign currency. The revenue functions, $r_y(y_h)$ and $R_y(y_f)$, are strictly increasing and concave.

The spot exchange rate at date t ($t = 1$ and 2), denoted by $\tilde{e}_t$ and expressed in units of the domestic currency against the foreign currency, is not known at $t = 0$.[8] We assume that the spot exchange rates follow a random walk so that $\tilde{e}_t = e_{t-1} + \tilde{\varepsilon}_t$, where $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ are two zero-mean random variables independent of each other.[9] To hedge the exchange rate risk, the MNF can trade infinitely divisible contracts in a currency futures market at $t = 0$. Each of the currency futures contracts calls for delivery of the domestic currency against the foreign currency at $t = 2$ and is marked to market at $t = 1$. The MNF is a price taker in the currency futures market. Let h be the number of the currency futures contracts sold (purchased if negative) by the MNF at $t = 0$.

[7] Broll and Zilcha (1992) and Lien and Wong (2005) consider an alternative case that the MNF's home and foreign operations produce the same homogeneous good. None of the qualitative results are affected if we follow their approach except that we would have one-way trade between the home and foreign operations rather than two-way trade as in our model. See also Meng and Wong (2007).

[8] Throughout the paper, random variables have a tilde ($\tilde{}$) while their realizations do not.

[9] An alternative way to model the exchange rate uncertainty is to apply the concept of information systems that are conditional cumulative distribution functions over a set of signals imperfectly correlated with $\tilde{e}_t$ (see Broll and Eckwert, 2006; Drees and Eckwert, 2003; Eckwert and Zilcha, 2001, 2003). The advantage of this more general and realistic approach is that one can study the value of information by comparing the information content of different information systems. Since the focus of this paper is not on the value of information, we adopt a simpler structure to save notation.

The futures exchange rate at date t ($t = 0$, 1, and 2) is denoted by f_t and expressed in units of the domestic currency per unit of the foreign currency. While the initial futures exchange rate, f_0, is predetermined at $t = 0$, the other futures exchange rates, $\tilde{f}_1$ and $\tilde{f}_2$, are regarded as random variables. By convergence, the futures exchange rate at $t = 2$ must be set equal to the spot exchange rate at that time. Thus, we have $\tilde{f}_2 = \tilde{e}_2$. To focus on the MNF's hedging motive, vis-à-vis its speculative motive, we assume that the currency futures market is intertemporally unbiased.[10] That is, we set the futures exchange rates at $t = 0$ and $t = 1$, f_0 and f_1, equal to the unconditional and conditional expected values of the spot exchange rate at $t = 2$, $\tilde{e}_2$, respectively. Given the random walk assumption about the spot exchange rate dynamics, the intertemporal unbiasedness of the currency futures market is tantamount to setting $f_0 = e_0$ and $\tilde{f}_1 = \tilde{e}_1$.

Conditional on the realized value of the futures exchange rate at $t = 1$, the MNF enjoys a net gain (or suffers a net loss if negative) from its futures position, h, equal to $(f_0 - f_1)h$. As in Lien (2003) and Wong (2004a, 2005), we assume that the MNF is liquidity constrained in that it is obliged to prematurely liquidate its futures position at $t = 1$ whenever the loss incurred at that time exceeds a prespecified threshold level, $k > 0$. Specifically, if $(f_1 - f_0)h > k$, the MNF prematurely liquidates its futures position at $t = 1$, implying that its random global profit at $t = 2$, denominated in the domestic currency, is given by

$$\begin{aligned}\tilde{\pi}_\ell &= r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) \\ &\quad + \tilde{e}_2[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] + (f_0 - f_1)h \\ &= r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) \\ &\quad + (e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] - \varepsilon_1 h, \end{aligned} \tag{1}$$

where the second equality follows from the intertemporal unbiasedness of the currency futures market and the random walk assumption about the spot exchange rate dynamics.

[10]If there are many risk-neutral speculators populated in the currency futures markets, the unbiasedness of the futures exchange rates is an immediate consequence of no arbitrage opportunities.

On the other hand, if $(f_1 - f_0)h \leq k$, the MNF holds its futures position until $t = 2$ so that its random global profit at $t = 2$ becomes

$$\tilde{\pi}_c = r_x(x_h) + r_y(y_h) - c_x(x_h + x_f)$$

$$+\tilde{e}_2[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] + (f_0 - \tilde{f}_2)h$$

$$= r_x(x_h) + r_y(y_h) - c_x(x_h + x_f)$$

$$+(e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] - (\varepsilon_1 + \tilde{\varepsilon}_2)h, \quad (2)$$

where the second equality follows from the intertemporal unbiasedness of the currency futures market and the random walk assumption about the spot exchange rate dynamics.

The MNF possesses a von Neumann-Morgenstern utility function, $u(\pi)$, defined over its global domestic currency profit at $t = 2$, π, with $u'(\pi) > 0$ and $u''(\pi) < 0$, indicating the presence of risk aversion.[11] The risk-averse behavior of the MNF can be motivated by managerial risk aversion (Stulz, 1984), corporate taxes (Smith and Stulz, 1985), costs of financial distress (Smith and Stulz, 1985), and/or capital market imperfections (Froot, Scharfstein, and Stein, 1993; Stulz, 1990).[12] Anticipating the liquidity constraint at $t = 1$, the MNF chooses its levels of sales in the home and foreign markets, x_h, y_h, x_f, and y_f, and selects its futures position, h, at $t = 0$ so as to maximize the expected utility of its random global domestic currency profit at $t = 2$:

$$EU = \begin{cases} \int_{-\infty}^{k/h} \mathrm{E}_2[u(\tilde{\pi}_c)]\,\mathrm{d}F(\varepsilon_1) + \int_{k/h}^{\infty} \mathrm{E}_2[u(\tilde{\pi}_\ell)]\,\mathrm{d}F(\varepsilon_1) & \text{if } h > 0, \\ \int_{-\infty}^{\infty} \mathrm{E}_2[u(\tilde{\pi}_0)]\,\mathrm{d}F(\varepsilon_1) & \text{if } h = 0, \\ \int_{-\infty}^{k/h} \mathrm{E}_2[u(\tilde{\pi}_\ell)]\,\mathrm{d}F(\varepsilon_1) + \int_{k/h}^{\infty} \mathrm{E}_2[u(\tilde{\pi}_c)]\,\mathrm{d}F(\varepsilon_1) & \text{if } h < 0, \end{cases} \quad (3)$$

where $\tilde{\pi}_\ell$ and $\tilde{\pi}_c$ are defined in Eqs. (1) and (2), respectively, $\tilde{\pi}_0$ is defined in Eq. (2) with $h = 0$, $F(\varepsilon_1)$ is the cumulative distribution function of $\tilde{\varepsilon}_1$, and $\mathrm{E}_2(\cdot)$ is the expectation operator with respect to the cumulative distribution function of $\tilde{\varepsilon}_2$.

[11] If the MNF is risk neutral, currency hedging adds no value to the MNF.

[12] See Tufano (1996) for evidence that managerial risk aversion is a rationale for corporate risk management in the gold mining industry.

2.1 Solution to the model

In order to solve the MNF's ex-ante decision problem, we need to know which expression on the right-hand side of Eq. (3) contains the solution. Consider first the case that $h > 0$. Using Leibniz's rule to partially differentiate EU as defined in Eq. (3) with respect to h and evaluating the resulting derivative at $h \to 0^+$ yields

$$\lim_{h\to 0^+} \frac{\partial EU}{\partial h} = -\int_{-\infty}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_0)(\varepsilon_1 + \tilde{\varepsilon}_2)] \, \mathrm{d}F(\varepsilon_1). \tag{4}$$

The right-hand side of Eq. (4) is simply the negative of the covariance between $u'(\tilde{\pi}_0)$ and $\tilde{\varepsilon}_1 + \tilde{\varepsilon}_2$. Since $\partial u'(\pi_0)/\partial(\varepsilon_1 + \varepsilon_2) = u''(\pi_0)[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] < 0$, we have $\lim_{h\to 0^+} \partial EU/\partial h > 0$. Now, consider the case where $h < 0$. Using Leibniz's rule to partially differentiate EU as defined in Eq. (3) with respect to h and evaluating the resulting derivative at $h \to 0^-$ yields

$$\lim_{h\to 0^-} \frac{\partial EU}{\partial h} = -\int_{-\infty}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_0)(\varepsilon_1 + \tilde{\varepsilon}_2)] \, \mathrm{d}F(\varepsilon_1). \tag{5}$$

Inspection of Eqs. (4) and (5) reveals that $\lim_{h\to 0^+} \partial EU/\partial h = \lim_{h\to 0^-} \partial EU/\partial h > 0$. The strict concavity of EU as defined in Eq. (3) implies that the MNF must optimally opt for $h > 0$.

Since the MNF's optimal futures position satisfies that $h > 0$, it follows from Eq. (3) that the MNF's ex-ante decision problem can be stated as

$$\max_{x_h, x_f, y_h, y_f, h} \int_{-\infty}^{k/h} \mathrm{E}_2[u(\tilde{\pi}_c)] \, \mathrm{d}F(\varepsilon_1) + \int_{k/h}^{\infty} \mathrm{E}_2[u(\tilde{\pi}_\ell)] \, \mathrm{d}F(\varepsilon_1). \tag{6}$$

The first-order conditions for program (6) with respect to x_h, x_f, y_h, y_f, and h are respectively given by

$$\int_{-\infty}^{k/h^*} \mathrm{E}_2[u'(\tilde{\pi}_c^*)][r_x'(x_h^*) - c_x'(x_h^* + x_f^*)] \, \mathrm{d}F(\varepsilon_1)$$

$$+ \int_{k/h^*}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)][r_x'(x_h^*) - c_x'(x_h^* + x_f^*)] \, \mathrm{d}F(\varepsilon_1) = 0, \tag{7}$$

$$\int_{-\infty}^{k/h^*} \mathrm{E}_2\{u'(\tilde{\pi}_c^*)[(e_0+\varepsilon_1+\tilde{\varepsilon}_2)R_x'(x_f^*) - c_x'(x_h^*+x_f^*)]\}\ \mathrm{d}F(\varepsilon_1)$$

$$+\int_{k/h^*}^{\infty} \mathrm{E}_2\{u'(\tilde{\pi}_\ell^*)[(e_0+\varepsilon_1+\tilde{\varepsilon}_2)R_x'(x_f^*) - c_x'(x_h^*+x_f^*)]\}\ \mathrm{d}F(\varepsilon_1) = 0, \tag{8}$$

$$\int_{-\infty}^{k/h^*} \mathrm{E}_2\{u'(\tilde{\pi}_c^*)[r_y'(y_h^*) - (e_0+\varepsilon_1+\tilde{\varepsilon}_2)c_y'(y_h^*+y_f^*)]\}\ \mathrm{d}F(\varepsilon_1)$$

$$+\int_{k/h^*}^{\infty} \mathrm{E}_2\{u'(\tilde{\pi}_\ell^*)[r_y'(y_h^*) - (e_0+\varepsilon_1+\tilde{\varepsilon}_2)c_y'(y_h^*+y_f^*)]\}\ \mathrm{d}F(\varepsilon_1) = 0, \tag{9}$$

$$\int_{-\infty}^{k/h^*} \mathrm{E}_2[u'(\tilde{\pi}_c^*)(e_0+\varepsilon_1+\tilde{\varepsilon}_2)][R_y'(y_f^*) - c_y'(y_h^*+y_f^*)]\ \mathrm{d}F(\varepsilon_1)$$

$$+\int_{k/h^*}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)(e_0+\varepsilon_1+\tilde{\varepsilon}_2)][R_y'(y_f^*) - c_y'(y_h^*+y_f^*)]\ \mathrm{d}F(\varepsilon_1) = 0, \tag{10}$$

and

$$-\int_{-\infty}^{k/h^*} \mathrm{E}_2[u'(\tilde{\pi}_c^*)(\varepsilon_1+\tilde{\varepsilon}_2)]\ \mathrm{d}F(\varepsilon_1) - \int_{k/h^*}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)]\varepsilon_1\ \mathrm{d}F(\varepsilon_1)$$

$$+\mathrm{E}_2[u(\tilde{\pi}_{\ell 1}^*) - u(\tilde{\pi}_{c1}^*)]F'(k/h^*)k/h^{*2} = 0, \tag{11}$$

where $\tilde{\pi}_{\ell 1}^*$ and $\tilde{\pi}_{c1}^*$ are defined in Eqs. (1) and (2) with $\varepsilon_1 = k/h^*$, respectively, Eq. (11) follows from Leibniz's rule, and an asterisk (*) signifies an optimal level.[13]

2.2 Benchmark without the liquidity constraint

As a benchmark, suppose that the MNF does not encounter the liquidity constraint, which is tantamount to setting $k = \infty$. In this benchmark case, Eqs. (7) to (11) reduce to

$$\int_{-\infty}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_c^0)][r_x'(x_h^0) - c_x'(x_h^0+x_f^0)]\ \mathrm{d}F(\varepsilon_1) = 0, \tag{12}$$

[13] The second-order conditions for program (6) are satisfied given risk aversion and the assumed properties of $r_x(x_h)$, $r_y(y_h)$, $R_x(x_f)$, $R_y(y_f)$, $c_x(x)$, and $c_y(y)$.

$$\int_{-\infty}^{\infty} \mathrm{E}_2\{u'(\tilde{\pi}_c^0)[(e_0+\varepsilon_1+\tilde{\varepsilon}_2)R_x'(x_f^0) - c_x'(x_h^0+x_f^0)]\}\,\mathrm{d}F(\varepsilon_1) = 0, \tag{13}$$

$$\int_{-\infty}^{\infty} \mathrm{E}_2\{u'(\tilde{\pi}_c^0)[r_y'(y_h^0) - (e_0+\varepsilon_1+\tilde{\varepsilon}_2)c_y'(y_h^0+y_f^0)]\}\,\mathrm{d}F(\varepsilon_1) = 0, \tag{14}$$

$$\int_{-\infty}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_c^0)(e_0+\varepsilon_1+\tilde{\varepsilon}_2)][R_y'(y_f^0) - c_y'(y_h^0+y_f^0)]\,\mathrm{d}F(\varepsilon_1) = 0, \tag{15}$$

and

$$-\int_{-\infty}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_c^0)(\varepsilon_1+\tilde{\varepsilon}_2)]\,\mathrm{d}F(\varepsilon_1) = 0, \tag{16}$$

where a nought (^{0}) indicates an optimal level in the absence of the liquidity constraint. Solving Eqs. (12) to (16) yields our first proposition.

Proposition 1. *Suppose that the risk-averse MNF has access to the intertemporally unbiased currency futures market for hedging purposes and is liquidity unconstrained, i.e., $k=\infty$. The MNF's optimal levels of sales in the home and foreign markets, x_h^0, x_f^0, y_h^0, and y_f^0, solve the following system of equations:*

$$r_x'(x_h^0) = c_x'(x_h^0+x_f^0), \tag{17}$$

$$e_0R_x'(x_f^0) = c_x'(x_h^0+x_f^0), \tag{18}$$

$$r_y'(y_h^0) = e_0c_y'(y_h^0+y_f^0), \tag{19}$$

and

$$R_y'(y_f^0) = c_y'(y_h^0+y_f^0), \tag{20}$$

and its optimal futures position, h^0, satisfies that $h^0 = R_x(x_f^0) + R_y(y_f^0) - c_y(y_h^0+y_f^0)$.

Proof. Eqs. (17) and (20) follow from Eqs. (12) and (15), respectively. Substituting Eq. (16) into Eqs. (13) and (14) yields Eqs. (18) and (19), respectively. Finally, if

$h^0 = R_x(x_f^0) + R_y(y_f^0) - c_y(y_h^0 + y_f^0)$, it follows from Eq. (2) that $\pi_c^0 = r_x(x_h^0) + r_y(y_h^0) - c_x(x_h^0 + x_f^0) + e_0[R_x(x_f^0) + R_y(y_f^0) - c_y(y_h^0 + y_f^0)]$, which is non-stochastic. Since $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ have means of zero, $h^0 = R_x(x_f^0) + R_y(y_f^0) - c_y(y_h^0 + y_f^0)$ indeed solves Eq. (16). This completes our proof. □

The results of Proposition 1 are simply the celebrated separation and full-hedging theorems emanated from the literature on the MNF under exchange rate uncertainty (see, e.g., Adam-Müller, 1997; Broll, 1992; Broll and Wong, 1999, 2006; Broll, Wong, and Zilcha, 1999; Wong, 2006). In the absence of the liquidity constraint, the MNF's random global domestic currency profit at $t = 2$ is given solely by Eq. (2), which can be written as

$$\tilde{\pi}_c = r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) + e_0[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$$

$$+(\varepsilon_1 + \tilde{\varepsilon}_2)[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f) - h]. \tag{21}$$

Since the currency futures market is intertemporally unbiased, it offers actuarially fair "insurance" to the MNF. The risk-averse MNF as such optimally opts for full insurance by adopting a full-hedge, i.e., $h = R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)$. Doing so completely eliminates the MNF's exposure to the exchange rate uncertainty, as is evident from Eq. (21). The MNF then optimally chooses it levels of sales in the home and foreign markets so as to maximize $r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) + e_0[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$, thereby yielding Eqs. (17) to (20). Eq. (17) states that the MNF equates the marginal cost of the good produced in the home operation to the marginal revenue of the good in the home market. Eqs. (17) and (18) imply that the marginal revenues of the good produced in the home operation, denominated in the domestic currency, are equalized in the home and foreign markets, where the exchange rate is locked in at the initial futures exchange rate, e_0. Likewise, Eqs. (19) and (20) imply similar optimality conditions for the good produced in the foreign operation.

3. Optimal hedging and sales decisions

We now resume our original case that the MNF encounters the liquidity constraint, i.e., $0 < k < \infty$. We shall contrast the MNF's optimal decisions with those in the benchmark case.

3.1 Optimal futures hedging decision

To examine the MNF's optimal futures hedging decision, we evaluate the left-hand side of Eq. (11) at $h^* = R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)$ to yield

$$-u'(\pi_c^*)\int_{-\infty}^{k/h^*}[\varepsilon_1 + \mathrm{E}_2(\tilde{\varepsilon}_2)]\,\mathrm{d}F(\varepsilon_1) - \mathrm{E}_2[u'(\pi_c^* + \tilde{\varepsilon}_2 h^*)]\int_{k/h^*}^{\infty}\varepsilon_1\,\mathrm{d}F(\varepsilon_1)$$

$$+\{\mathrm{E}_2[u(\pi_c^* + \tilde{\varepsilon}_2 h^*)] - u(\pi_c^*)\}F'(k/h^*)k/h^{*2}, \tag{22}$$

where $\pi_c^* = r_x(x_h^*) + r_y(y_h^*) - c_x(x_h^* + x_f^*) + e_0 h^*$. Since $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ have means of zero, we can write expression (22) as

$$\{u'(\pi_c^*) - \mathrm{E}_2[u'(\pi_c^* + \tilde{\varepsilon}_2 h^*)]\}\int_{k/h^*}^{\infty}\varepsilon_1\,\mathrm{d}F(\varepsilon_1)$$

$$+\{\mathrm{E}_2[u(\pi_c^* + \tilde{\varepsilon}_2 h^*)] - u(\pi_c^*)\}F'(k/h^*)k/h^{*2}. \tag{23}$$

Since $\mathrm{E}_2(\tilde{\varepsilon}_2) = 0$, it follows from risk aversion and Jensen's inequality that $u(\pi_c^*) > \mathrm{E}_2[u(\pi_c^* + \tilde{\varepsilon}_2 h^*)]$ and thus the second term of expression (23) is unambiguously negative. The first term of expression (23) is, however, indeterminate without knowing the sign of $u'''(\pi)$.

As convincingly argued by Kimball (1990, 1993), prudence, i.e., $u'''(\pi) \geq 0$, is a reasonable behavioral assumption for decision making under multiple sources of uncertainty. Prudence measures the propensity to prepare and forearm oneself under uncertainty, vis-à-vis risk aversion that is how much one dislikes uncertainty and would turn away from it if one could. As shown by Drèze and Modigliani (1972), Kimball (1990), and Leland (1968), prudence is both necessary and sufficient to induce precautionary saving. Furthermore, prudence is implied by decreasing absolute risk aversion, which is instrumental in yielding many intuitively appealing comparative statics under uncertainty (Gollier, 2001).

Given prudence, it follows from $\mathrm{E}_2(\tilde{\varepsilon}_2) = 0$ and Jensen's inequality that $u'(\pi_c^*) \leq \mathrm{E}_2[u'(\pi_c^* + \tilde{\varepsilon}_2 h^*)]$, where the equality holds only when $u'''(\pi) \equiv 0$. Since $\tilde{\varepsilon}_1$ has a mean of zero, the first-term of expression (23) is non-positive, thereby implying that expression (23) is also non-positive, should $u'''(\pi) \geq 0$. The following proposition is then an immediate consequence of Eq. (11) and the second-order conditions of program (6).

Proposition 2. *Suppose that the risk-averse MNF has access to the intertemporally unbiased currency futures market for hedging purposes and is liquidity constrained, i.e.,* $0 < k < \infty$*. If the MNF is prudent, i.e.,* $u'''(\pi) \geq 0$*, its optimal futures position,* h^**, satisfies that* $0 < h^* < R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)$*.*

The intuition of Proposition 2 is as follows. If the MNF does not encounter the liquidity constraint, we have shown in Proposition 1 that the full-hedging theorem applies in that setting $h = R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)$ completely eliminates the exchange rate risk. In the presence of the liquidity constraint, however, such a full-hedge is no longer optimal for the MNF due to the residual risk, $\tilde{\varepsilon}_2[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$, arising from the premature liquidation of its futures position at $t = 1$, as is evident from Eq. (1). If the MNF has a quadratic utility function, it is well-known that its optimal futures position is the one that minimizes the variability of its random global domestic currency profit at $t = 2$. Since the residual risk due to the premature liquidation of the futures position at $t = 1$ prevails for all $\varepsilon_1 > k/h$, the MNF has incentives to shrink this interval by setting h below $R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)$. As such, the liquidity constrained MNF finds an under-hedge, i.e., $h^* < R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)$, optimal when its utility function is quadratic.

Given prudence, the MNF is more sensitive to low realizations of its random global domestic currency profit at $t = 2$ than to high ones (Kimball, 1990, 1993). Note that the low realizations of its random global domestic currency profit at $t = 2$ occur when the MNF prematurely liquidates its futures position at $t = 1$ and the realized values of $\tilde{\varepsilon}_1$ are negative. Inspection of Eqs. (1) and (2) reveals that the MNF can avoid these realizations by shorting less of the currency futures contracts. Thus, the under-hedging incentive as described under quadratic utility functions is reinforced when the MNF becomes prudent in the sense of Kimball (1990, 1993). It is therefore optimal for the prudent MNF to set $h^* < R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)$ to strike a balance between the extent of the exchange rate risk and that of the liquidity risk.[14]

Tufano (1996) documents that in the gold mining industry only 17% of firms shed 40% or more of their price risk. Bodnar, Hayt, and Marston (1998) also find that most non-financial firms do not use financial derivatives to completely hedge their risk exposures. Such a normal practice of partial hedging may be due to the prevalence of liquidity constraints confronted by firms, as suggested by Proposition 2.

3.2 Optimal production and sales decisions

To examine the optimal production and sales decisions of the MNF in the presence of the liquidity constraint, we solve Eqs. (7) to (11) to yield the following proposition

Proposition 3. *Suppose that the risk-averse MNF has access to the intertemporally unbiased currency futures market for hedging purposes and is liquidity constrained, i.e.,* $0 < k < \infty$. *The MNF's optimal levels of sales in the home and foreign markets,* x_h^*, x_f^*, y_h^*, *and* y_f^*, *solve the following system of equations:*

$$r_x'(x_h^*) = c_x'(x_h^* + x_f^*), \tag{24}$$

[14] Tailing has the similar effect on inducing hedgers to use fewer futures contracts initially than they would if forward contracts were used. A tail alters a futures position in a way that the adjusted futures position behaves exactly like a forward position. The tailing adjustment is greater the longer the position to be held and the higher the riskless rate of interest (Figlewski, Lanskroner, and Silber, 1991).

$$(e_0 - \theta)R'_x(x_f^*) = c'_x(x_h^* + x_f^*), \tag{25}$$

$$r'_y(y_h^*) = (e_0 - \theta)c'_y(y_h^* + y_f^*), \tag{26}$$

and

$$R'_y(y_f^*) = c'_y(y_h^* + y_f^*), \tag{27}$$

where

$$\theta = \frac{\mathrm{E}_2[u(\tilde{\pi}_{c1}^*) - u(\tilde{\pi}_{\ell 1}^*)]F'(k/h^*)k/h^{*2} - \int_{k/h^*}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)\tilde{\varepsilon}_2]\ \mathrm{d}F(\varepsilon_1)}{\int_{-\infty}^{k/h^*} \mathrm{E}_2[u'(\tilde{\pi}_c^*)]\ \mathrm{d}F(\varepsilon_1) + \int_{k/h^*}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)]\ \mathrm{d}F(\varepsilon_1)}. \tag{28}$$

Furthermore, $\theta > 0$ if the MNF is prudent.

Proof. Eqs. (24) and (27) follow from Eqs. (7) and (10), respectively. Substituting Eq. (11) into Eqs. (8) and (9) yields Eqs. (25) and (26), respectively. Since $\tilde{\varepsilon}_2$ has a mean of zero, $\mathrm{E}_2[u'(\tilde{\pi}_\ell^*)\tilde{\varepsilon}_2]$ is simply the covariance between $u'(\tilde{\pi}_\ell^*)$ and $\tilde{\varepsilon}_2$. Since $\partial u'(\pi_\ell^*)/\partial\varepsilon_2 = u''(\pi_\ell^*)[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)] < 0$, we have $\mathrm{E}_2[u'(\tilde{\pi}_\ell^*)\tilde{\varepsilon}_2] < 0$ for all $\varepsilon_1 > k/h^*$. To show that $\theta > 0$ should the MNF be prudent, it suffices to show that $\mathrm{E}_2[u(\tilde{\pi}_{c1}^*)] > \mathrm{E}_2[u(\tilde{\pi}_{\ell 1}^*)]$ if $u'''(\pi) \geq 0$, as is evident from Eq. (28).

Let $q^* = R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)$ and $\bar{\pi}^* = r_x(x_h^*) + r_y(y_h^*) - c_x(x_h^* + x_f^*) + (e_0 + k/h^*)q^* - k$. Note that $\mathrm{E}_2(\tilde{\pi}_{c1}^*) = \mathrm{E}_2(\tilde{\pi}_{\ell 1}^*) = \bar{\pi}^*$. Hence, $\tilde{\pi}_{c1}^*$ and $\tilde{\pi}_{\ell 1}^*$ have the same mean, $\bar{\pi}^*$. From Proposition 2, we know that $0 < h^* \leq q^*$ given prudence, where the equality holds only when $k = 0$ and $u'''(\pi) \equiv 0$. If $h^* = q^*$, we have $\tilde{\pi}_{c1}^* \equiv \bar{\pi}^*$. Thus, when $k = 0$ and $u'''(\pi) \equiv 0$, it follows from risk aversion that $\mathrm{E}_2[u(\tilde{\pi}_{c1}^*)] = u(\bar{\pi}^*) > \mathrm{E}_2[u(\tilde{\pi}_{\ell 1}^*)]$. On the other hand, if $h^* < q^*$, we let $\Psi(\pi_{c1}^*)$ be the cumulative distribution function of $\tilde{\pi}_{c1}^*$. Using the change-of-variable technique (Hogg and Craig, 1989), we have $\Psi(\pi_{c1}^*) = F[(\pi_{c1}^* - \bar{\pi}^*)/(q^* - h^*)]$. Likewise, we let $\Phi(\pi_{\ell 1}^*)$ be the cumulative distribution function of $\tilde{\pi}_{\ell 1}^*$. Using the change-of-variable technique, we have $\Phi(\pi_{\ell 1}^*) = F[(\pi_{\ell 1}^* - \bar{\pi}^*)/q^*]$. Consider the function, $T(\pi) = \int_{-\infty}^{\pi}[\Phi(z) - \Psi(z)]\ \mathrm{d}z$. Using Leibniz's rule, we have $T'(\pi) = \Phi(\pi) - \Psi(\pi) = F[(\pi - \bar{\pi}^*)/q^*] - F[(\pi - \bar{\pi}^*)/(q^* - h^*)] > (<)\ 0$ for all $\pi < (>)\ \bar{\pi}^*$. Note that $T(-\infty) = 0$

and that $T(\infty) = 0$ because $\tilde{\pi}^*_{c1}$ and $\tilde{\pi}^*_{\ell 1}$ have the same mean, $\bar{\pi}^*$. Since $T(\pi)$ is strictly increasing (decreasing) for all $\pi < (>)\ \bar{\pi}^*$, we conclude that $T(\pi) > 0$ for all π. In other words, $\Phi(\pi)$ is a mean preserving spread of $\Psi(\pi)$ in the sense of Rothschild and Stiglitz (1970). It then follows from Rothschild and Stiglitz (1971) that $\mathrm{E}_2[u(\tilde{\pi}^*_{c1})] > \mathrm{E}_2[u(\tilde{\pi}^*_{\ell 1})]$ given risk aversion. □

Comparing the set of optimality conditions with the liquidity constraint, Eqs. (24) to (27), to that without the liquidity constraint, Eqs. (17) to (20), yields the following proposition.

Proposition 4. *Suppose that the risk-averse MNF has access to the intertemporally unbiased currency futures market for hedging purposes. If the MNF is prudent, imposing the liquidity constraint, i.e., $0 < k < \infty$, on the MNF induces (i) greater sales of both goods in the home market, i.e., $x^*_h > x^0_h$ and $y^*_h > y^0_h$, (ii) lower sales of both goods in the foreign market, i.e., $x^*_f < x^0_f$ and $y^*_f < y^0_f$, (iii) lower output in the home operation, i.e., $x^*_h + x^*_f < x^0_h + x^0_f$, and (iv) higher output in the foreign operation, i.e., $y^*_h + y^*_f > y^0_h + y^0_f$.*

Proof. Suppose first that $x^*_h \leq x^0_h$. Since $r''_x(x_h) < 0$, we have $r'_x(x^*_h) \geq r'_x(x^0_h)$. Eqs. (17) and (24) then imply that $c'_x(x^*_h + x^*_f) \geq c'_x(x^0_h + x^0_f)$. Since $c''_x(x) > 0$ and $x^*_h \leq x^0_h$, we must have $x^*_f \geq x^0_f$. From Proposition 3, $\theta > 0$ if the MNF is prudent. It then follows from $R''_x(x_f) < 0$ that $(f_0 - \theta)R'_x(x^*_f) < f_0 R'_x(x^0_f)$. But this inequality together with Eqs. (18) and (25) would imply that $c'_x(x^*_h + x^*_f) < c'_x(x^0_h + x^0_f)$, a contradiction. Hence, the supposition is not true and we must have $x^*_h > x^0_h$. It then follows from $r''_x(x_h) < 0$ that $r'_x(x^*_h) < r'_x(x^0_h)$. Eqs. (17) and (24) thus imply that $c'_x(x^*_h + x^*_f) < c'_x(x^0_h + x^0_f)$. Since $c''_x(x) > 0$, we must have $x^*_h + x^*_f < x^0_h + x^0_f$ and thereby $x^*_f < x^0_f$.

Now, suppose that $y^*_f \geq y^0_f$. Since $R''_y(y_f) < 0$, we have $R'_y(y^*_f) \leq R'_y(y^0_f)$. Eqs. (20) and (27) then imply that $c'_y(y^*_h + y^*_f) \leq c'_y(y^0_h + y^0_f)$. Since $c''_y(y) > 0$ and $y^*_f \geq y^0_f$, we must have $y^*_h \leq y^0_h$. It then follows from $r''_y(y_h) < 0$ that $r'_y(y^*_h) \geq r'_y(y^0_h)$. But this inequality together with Eqs. (18) and (25) and $\theta > 0$ would imply that $c'_y(y^*_h + y^*_f) > c'_y(y^0_h + y^0_f)$, a contradiction. Hence, the supposition is not true and we must have $y_f^* < y_f^0$. It then follows from $R_y''(y_f) < 0$ that $R_y'(y_f^*) > R_y'(y_f^0)$. Eqs. (20) and (27) thus imply that $c_y'(y_h^* + y_f^*) > c_y'(y_h^0 + y_f^0)$. Since $c_y''(y) > 0$, we must have $y_h^* + y_f^* > y_h^0 + y_f^0$ and thereby $y_h^* > y_h^0$. □

The intuition of Proposition 4 is as follows. In the presence of the liquidity constraint, setting $h = R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)$ cannot eliminate all the exchange rate risk due to the residual risk, $\tilde{\varepsilon}_2[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$, arising from the premature liquidation of the futures position at $t = 1$, as is evident from Eq. (1). Such residual risk, however, can be controlled by varying the levels of sales in the home and foreign markets. Eq. (24) states that it remains optimal for the MNF to equate the marginal cost of the good produced in the home operation to the marginal revenue of the good in the home market. Eqs. (24) and (25), however, imply that the marginal revenue of the good produced in the home operation is strictly smaller in the home market than in the foreign market, where the latter is denominated in the domestic currency with the exchange rate locked in at the initial futures exchange rate, e_0. Since the sales in the foreign market are embedded with some exchange rate risk that cannot be eliminated due to the presence of the liquidity constraint, the MNF has to demand a risk premium to compensate for its foreign sales. The wedge between the two marginal revenues in the home and foreign markets is *de facto* the risk premium required by the MNF. Similar arguments apply to the good produced in the foreign operation. The MNF as such sells less (more) and produces more (less) in the foreign (home) country. These results are in line with the findings of Broll and Zilcha (1992), Lien and Wong (2005), and Meng and Wong (2007).

In the absence of the liquidity constraint, the MNF optimally adopts a full-hedge that eliminates all the exchange rate risk. The MNF then chooses its sales and outputs so as to maximize its expected global domestic currency profit at $t = 2$. In the presence of the liquidity constraint, the MNF adjusts its sales and outputs according to Proposition 4 by selling less (more) and producing more (less) in the foreign (home) country. This must imply the MNF's expected global domestic currency profit at $t = 2$ be adversely affected in face of the liquidity risk. Given risk aversion, the MNF's expected utility must also be lowered. Thus, the presence of the liquidity constraint unambiguously makes the MNF strictly worse off regarding to both its expected profit and utility levels.

4. Hedging role of futures spreads

Thus far, we have restricted the MNF to use the currency futures contracts that mature at $t = 2$ for hedging purposes. In this section, we introduce another type of currency futures contracts that mature at $t = 1$ to the MNF. We refer to the former as the distant futures contracts and the latter as the nearby futures contracts. Both types of contracts are infinitely divisible and tradable in the currency futures market at $t = 0$. The MNF is a price taker in the currency futures market.

Each of the nearby futures contracts calls for delivery of the domestic currency against the foreign currency at $t = 1$. Denote ${}_tf_1$ as the futures exchange rate of the nearby contracts at date t ($t = 0$ and 1). On the other hand, each of the distant futures contracts calls for delivery of the domestic currency against the foreign currency at $t = 2$ and is marked to market at $t = 1$. Denote ${}_tf_2$ as the futures exchange rate of the distant contracts at date t ($t = 0$, 1, and 2). While the initial futures exchange rates, ${}_0f_1$ and ${}_0f_2$, are predetermined at $t = 0$, all other futures exchange rates, ${}_1\tilde{f}_1$, ${}_1\tilde{f}_2$, and ${}_2\tilde{f}_2$, are regarded as random variables. Let h_1 and h_2 be the numbers of the nearby and distant futures contracts sold (purchased if negative) by the MNF at $t = 0$.

By convergence, the futures exchange rate of the nearby contracts at the maturity date must be set equal to the spot exchange rate at $t = 1$ so that ${}_1\tilde{f}_1 = \tilde{e}_1$. Likewise, the futures exchange rate of the distant contracts at the maturity date must be set equal to the spot exchange rate at $t = 2$ so that ${}_2\tilde{f}_2 = \tilde{e}_2$. To focus on the MNF's hedging motive, we assume away any biasedness and spread basis in the currency futures market. The absence of spread basis allows us to write $f_0 = {}_0f_1 = {}_0f_2$ and $f_1 = {}_1f_1 = {}_1f_2$. Unbiasedness implies that f_0

and f_1 are set equal to the unconditional and conditional expectations of the spot exchange rate at $t = 2$, $\tilde{e}_2$, respectively, which coupled with the random walk assumption about the spot exchange rate dynamics is equivalent to setting $f_0 = e_0$ and $\tilde{f}_1 = \tilde{e}_1$.

Conditioned on the realized futures exchange rates of the nearby and distant futures contracts at $t = 1$, ${}_1f_1$ and ${}_1f_2$, the MNF enjoys a net gain (or suffers a net loss if negative) from its nearby and distant futures positions at $t = 1$ equal to $({}_0f_1 - {}_1f_1)h_1 + ({}_0f_2 - {}_1f_2)h_2$. The MNF is liquidity constrained in that it is obliged to prematurely liquidate its distant futures position at $t = 1$ whenever the net loss incurred at that time exceeds the prespecified threshold level, $k > 0$. Specifically, if $({}_1f_1 - {}_0f_1)h_1 + ({}_1f_2 - {}_0f_2)h_2 > k$, the MNF prematurely liquidates its distant futures position at $t = 1$, implying that its random global domestic currency profit at $t = 2$ is given by

$$\begin{aligned}\tilde{\pi}_\ell &= r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) \\ &\quad + \tilde{e}_2[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] + ({}_0f_1 - {}_1f_1)h_1 + ({}_0f_2 - {}_1f_2)h_2 \\ &= r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) \\ &\quad + (e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] - \varepsilon_1(h_1 + h_2), \end{aligned} \tag{29}$$

where the second equality follows from the intertemporal unbiasedness of the currency futures market and the random walk assumption about the spot exchange rate dynamics. On the other hand, if $({}_1f_1 - {}_0f_1)h_1 + ({}_1f_2 - {}_0f_2)h_2 \leq k$, the MNF holds its distant futures position until $t = 2$ so that its random global domestic currency profit at $t = 2$ becomes

$$\begin{aligned}\tilde{\pi}_c &= r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) \\ &\quad + \tilde{e}_2[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] + ({}_0f_1 - {}_1f_1)h_1 + ({}_0f_2 - {}_2\tilde{f}_2)h_2 \\ &= r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) \\ &\quad + (e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] - \varepsilon_1(h_1 + h_2) - \tilde{\varepsilon}_2 h_2, \end{aligned} \tag{30}$$

where the second equality follows from the intertemporal unbiasedness of the currency futures market and the random walk assumption about the spot exchange rate dynamics.

Anticipating the liquidity constraint at $t = 1$, the MNF chooses its levels of sales in the home and foreign markets, x_h, y_h, x_f, and y_f, and selects its nearby futures position, h_1, and its distant futures position, h_2, at $t = 0$ so as to maximize the expected utility of its random global domestic currency profit at $t = 2$:

$$EU = \begin{cases} \int_{-\infty}^{k/(h_1+h_2)} \mathrm{E}_2[u(\tilde{\pi}_c)]\ \mathrm{d}F(\varepsilon_1) + \int_{k/(h_1+h_2)}^{\infty} \mathrm{E}_2[u(\tilde{\pi}_\ell)]\ \mathrm{d}F(\varepsilon_1) & \text{if } h_1 + h_2 > 0, \\ \int_{-\infty}^{\infty} \mathrm{E}_2[u(\tilde{\pi}_0)]\ \mathrm{d}F(\varepsilon_1) & \text{if } h_1 + h_2 = 0, \\ \int_{-\infty}^{k/(h_1+h_2)} \mathrm{E}_2[u(\tilde{\pi}_\ell)]\ \mathrm{d}F(\varepsilon_1) + \int_{k/(h_1+h_2)}^{\infty} \mathrm{E}_2[u(\tilde{\pi}_c)]\ \mathrm{d}F(\varepsilon_1) & \text{if } h_1 + h_2 < 0, \end{cases} \quad (31)$$

where $\tilde{\pi}_\ell$ and $\tilde{\pi}_c$ are defined in Eqs. (29) and (30), respectively, and $\tilde{\pi}_0$ is defined in Eq. (30) with $h_1 + h_2 = 0$.

In order to solve the MNF's ex-ante decision problem, we need to know which expression on the right-hand side of Eq. (31) contains the solution. To this end, let $\gamma = h_1 + h_2$. Consider first the case that $\gamma > 0$. Using Leibniz's rule to partially differentiate EU as defined in Eq. (31) with respect to γ and evaluating the resulting derivative at $\gamma \to 0^+$ yields

$$\lim_{\gamma \to 0^+} \frac{\partial EU}{\partial \gamma} = -\int_{-\infty}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_0)]\varepsilon_1\ \mathrm{d}F(\varepsilon_1). \quad (32)$$

The right-hand side of Eq. (32) is simply the negative of the covariance between $\mathrm{E}_2[u'(\tilde{\pi}_0)]$ and $\tilde{\varepsilon}_1$. Since $\partial \mathrm{E}_2[u'(\tilde{\pi}_0)]/\partial \varepsilon_1 = \mathrm{E}_2[u''(\tilde{\pi}_0)][R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)] < 0$, we have $\lim_{\gamma \to 0^+} \partial EU/\partial \gamma > 0$. Now, consider the case that $\gamma < 0$. Using Leibniz's rule to partially differentiate EU as defined in Eq. (31) with respect to γ and evaluating the resulting derivative at $\gamma \to 0^-$ yields

$$\lim_{\gamma \to 0^-} \frac{\partial EU}{\partial \gamma} = -\int_{-\infty}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_0)]\varepsilon_1\ \mathrm{d}F(\varepsilon_1). \quad (33)$$

Inspection of Eqs. (32) and (33) reveals that $\lim_{\gamma \to 0^+} \partial EU/\partial \gamma = \lim_{\gamma \to 0^-} \partial EU/\partial \gamma > 0$. The strict concavity of EU as defined in Eq. (31) implies that the MNF must optimally opt for $h_1 + h_2 > 0$.

4.1 Optimal hedging and sales decisions

Since the MNF's optimal nearby and distant futures positions satisfy that $h_1 + h_2 > 0$, it follows from Eq. (31) that the MNF's ex-ante decision problem is given by

$$\max_{x_h, x_f, y_h, y_f, h_1, h_2} \int_{-\infty}^{k/(h_1+h_2)} \mathrm{E}_2[u(\tilde{\pi}_c)]\ \mathrm{d}F(\varepsilon_1) + \int_{k/(h_1+h_2)}^{\infty} \mathrm{E}_2[u(\tilde{\pi}_\ell)]\ \mathrm{d}F(\varepsilon_1). \tag{34}$$

The first-order conditions for program (34) with respect to x_h, x_f, y_h, y_f, h_1, and h_2 are respectively given by

$$\int_{-\infty}^{k/(h_1^*+h_2^*)} \mathrm{E}_2[u'(\tilde{\pi}_c^*)][r_x'(x_h^*) - c_x'(x_h^* + x_f^*)]\ \mathrm{d}F(\varepsilon_1)$$

$$+\int_{k/(h_1^*+h_2^*)}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)][r_x'(x_h^*) - c_x'(x_h^* + x_f^*)]\ \mathrm{d}F(\varepsilon_1) = 0, \tag{35}$$

$$\int_{-\infty}^{k/(h_1^*+h_2^*)} \mathrm{E}_2\{u'(\tilde{\pi}_c^*)[(e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)R_x'(x_f^*) - c_x'(x_h^* + x_f^*)]\}\ \mathrm{d}F(\varepsilon_1)$$

$$+\int_{k/(h_1^*+h_2^*)}^{\infty} \mathrm{E}_2\{u'(\tilde{\pi}_\ell^*)[(e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)R_x'(x_f^*) - c_x'(x_h^* + x_f^*)]\}\ \mathrm{d}F(\varepsilon_1) = 0, \tag{36}$$

$$\int_{-\infty}^{k/(h_1^*+h_2^*)} \mathrm{E}_2\{u'(\tilde{\pi}_c^*)[r_y'(y_h^*) - (e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)c_y'(y_h^* + y_f^*)]\}\ \mathrm{d}F(\varepsilon_1)$$

$$+\int_{k/(h_1^*+h_2^*)}^{\infty} \mathrm{E}_2\{u'(\tilde{\pi}_\ell^*)[r_y'(y_h^*) - (e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)c_y'(y_h^* + y_f^*)]\}\ \mathrm{d}F(\varepsilon_1) = 0, \tag{37}$$

$$\int_{-\infty}^{k/(h_1^*+h_2^*)} \mathrm{E}_2[u'(\tilde{\pi}_c^*)(e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)][R_y'(y_f^*) - c_y'(y_h^* + y_f^*)]\ \mathrm{d}F(\varepsilon_1)$$

$$+\int_{k/(h_1^*+h_2^*)}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)(e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)][R_y'(y_f^*) - c_y'(y_h^* + y_f^*)]\ \mathrm{d}F(\varepsilon_1) = 0, \tag{38}$$

$$-\int_{-\infty}^{k/(h_1^*+h_2^*)} \mathrm{E}_2[u'(\tilde{\pi}_c^*)]\varepsilon_1\ \mathrm{d}F(\varepsilon_1) - \int_{k/(h_1^*+h_2^*)}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)]\varepsilon_1\ \mathrm{d}F(\varepsilon_1)$$

$$+\mathrm{E}_2[u(\tilde{\pi}_{\ell 1}^*) - u(\tilde{\pi}_{c1}^*)]F'[k/(h_1^* + h_2^*)]k/(h_1^* + h_2^*)^2 = 0, \tag{39}$$

and

$$-\int_{-\infty}^{k/(h_1^*+h_2^*)} \mathrm{E}_2[u'(\tilde{\pi}_c^*)(\varepsilon_1+\tilde{\varepsilon}_2)]\ \mathrm{d}F(\varepsilon_1) - \int_{k/(h_1^*+h_2^*)}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)]\varepsilon_1\ \mathrm{d}F(\varepsilon_1)$$

$$+\mathrm{E}_2[u(\tilde{\pi}_{\ell 1}^*) - u(\tilde{\pi}_{c1}^*)]F'[k/(h_1^*+h_2^*)]k/(h_1^*+h_2^*)^2 = 0, \tag{40}$$

where $\tilde{\pi}_{\ell 1}^*$ and $\tilde{\pi}_{c1}^*$ are defined in Eqs. (29) and (30) with $\varepsilon_1 = k/(h_1^*+h_2^*)$, respectively, Eqs. (39) and (40) follow from Leibniz's rule, and an asterisk (*) signifies an optimal level. Solving Eqs. (35) to (40) yields the following proposition.

Proposition 5. *Suppose that the risk-averse MNF has access to the intertemporally unbiased currency futures market for hedging purposes and is liquidity constrained, i.e.,* $0 < k < \infty$*. The MNF's optimal nearby and distant futures positions,* h_1^* *and* h_2^**, satisfy that* $h_1^* + h_2^* > 0$ *and* $h_2^* = R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)$*. The MNF's optimal levels of sales in the home and foreign markets,* x_h^**,* x_f^**,* y_h^**, and* y_f^**, solve the following system of equations:*

$$r_x'(x_h^*) = c_x'(x_h^* + x_f^*), \tag{41}$$

$$(e_0 - \phi)R_x'(x_f^*) = c_x'(x_h^* + x_f^*), \tag{42}$$

$$r_y'(y_h^*) = (e_0 - \phi)c_y'(y_h^* + y_f^*), \tag{43}$$

and

$$R_y'(y_f^*) = c_y'(y_h^* + y_f^*), \tag{44}$$

where $\pi_c^* = r_x(x_h^*) + r_y(y_h^*) - c_x(x_h^* + x_f^*) + e_0 h_2^* - \varepsilon_1 h_1^*$*,* $\tilde{\pi}_\ell^* = \pi_c^* + \tilde{\varepsilon}_2 h_2^*$*,* $\pi_{c1}^* = r_x(x_h^*) + r_y(y_h^*) - c_x(x_h^* + x_f^*) + e_0 h_2^* - kh_1^*/(h_1^* + h_2^*)$*,* $\tilde{\pi}_{\ell 1}^* = \pi_{c1}^* + \tilde{\varepsilon}_2 h_2^*$*, and*

$$\phi = \frac{\{u(\pi_{c1}^*) - \mathrm{E}[u(\tilde{\pi}_{\ell 1}^*)]\}F'[k/(h_1^*+h_2^*)]k/(h_1^*+h_2^*)^2 - \int_{k/(h_1^*+h_2^*)}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)\tilde{\varepsilon}_2]\ \mathrm{d}F(\varepsilon_1)}{\int_{-\infty}^{k/(h_1^*+h_2^*)} u'(\pi_c^*)\ \mathrm{d}F(\varepsilon_1) + \int_{k/(h_1^*+h_2^*)}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)]\ \mathrm{d}F(\varepsilon_1)}, \tag{45}$$

which is strictly positive.

Proof. Subtracting Eq. (39) by Eq. (40) yields

$$\int_{-\infty}^{k/(h_1^*+h_2^*)} \mathrm{E}_2[u'(\tilde{\pi}_c^*)\tilde{\varepsilon}_2]\,\mathrm{d}F(\varepsilon_1) = 0. \tag{46}$$

Since $\mathrm{E}_2(\tilde{\varepsilon}_2) = 0$, $\mathrm{E}_2[u'(\tilde{\pi}_c^*)\tilde{\varepsilon}_2]$ is simply the covariance between $u'(\tilde{\pi}_c^*)$ and $\tilde{\varepsilon}_2$. Note that $\partial u'(\pi_c^*)/\partial\varepsilon_2 = u''(\pi_c^*)[R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*) - h_2^*]$. If $h_2^* < (>)\ R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)$, we have $\mathrm{E}_2[u'(\tilde{\pi}_c^*)\tilde{\varepsilon}_2] < (>)\ 0$ for all $\varepsilon_1 \leq k/(h_1^* + h_2^*)$, a contradiction to Eq. (46). Thus, it must be true that $h_2^* = R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)$.

Eqs. (41) and (44) follow from Eqs. (35) and (38), respectively. Substituting Eq. (40) into Eqs. (36) and (37) yields Eqs. (42) and (43), respectively. Since $\tilde{\varepsilon}_2$ has a mean of zero, $\mathrm{E}_2[u'(\tilde{\pi}_\ell^*)\tilde{\varepsilon}_2]$ is simply the covariance between $u'(\tilde{\pi}_\ell^*)$ and $\tilde{\varepsilon}_2$. Since $\partial u'(\pi_\ell^*)/\partial\varepsilon_2 = u''(\pi_\ell^*)h_2^* < 0$, we have $\mathrm{E}_2[u'(\tilde{\pi}_\ell^*)\tilde{\varepsilon}_2] < 0$ for all $\varepsilon_1 > k/(h_1^* + h_2^*)$. Since $\tilde{\pi}_{\ell 1}^* = \pi_{c1}^* + \tilde{\varepsilon}_2 h_2^*$ and $\mathrm{E}_2(\tilde{\varepsilon}_2) = 0$, it follows from risk aversion and Jensen's inequality that $u(\pi_{c1}^*) > \mathrm{E}_2[u(\tilde{\pi}_{\ell 1}^*)]$. Thus, Eq. (45) implies that $\phi > 0$. □

Comparing the set of optimality conditions with the liquidity constraint, Eqs. (41) to (44), to that without the liquidity constraint, Eqs. (17) to (20), yields the following proposition.

Proposition 6. *Suppose that the risk-averse MNF has access to the intertemporally unbiased currency futures market for hedging purposes. Imposing the liquidity constraint, i.e., $0 < k < \infty$, on the MNF induces (i) greater sales of both goods in the home market, i.e., $x_h^* > x_h^0$ and $y_h^* > y_h^0$, (ii) lower sales of both goods in the foreign market, i.e., $x_f^* < x_f^0$ and $y_f^* < y_f^0$, (iii) lower output in the home operation, i.e., $x_h^* + x_f^* < x_h^0 + x_f^0$, and (iv) higher output in the foreign operation, i.e., $y_h^* + y_f^* > y_h^0 + y_f^0$.*

Proof. The proof is identical to that of Proposition 4 and thus is omitted. □

In Proposition 4 where the MNF is restricted to use the distant futures contracts as the sole hedging instrument, prudence is called for to yield the intuitive effects of the liquidity constraint on production and sales . When the MNF can use both the nearby and distant futures contracts, the hedging environment becomes less incomplete such that risk aversion alone is enough to guarantee the same intuitive effects of the liquidity constraint on production and sales.

4.2 Optimality of futures spreads

While Proposition 5 explicitly characterizes the MNF's optimal distant futures position, h_2^*, it is silent about the MNF's optimal nearby futures position, h_1^*. To examine h_1^*, we evaluate the left-hand side of Eq. (39) at $h_1^* = 0$ to yield

$$-u'(\pi_c^*)\int_{-\infty}^{k/h_2^*}\varepsilon_1\ \mathrm{d}F(\varepsilon_1) - \mathrm{E}_2[u'(\pi_c^* + \tilde{\varepsilon}_2 h_2^*)]\int_{k/h_2^*}^{\infty}\varepsilon_1\ \mathrm{d}F(\varepsilon_1)$$
$$+\{\mathrm{E}_2[u(\pi_c^* + \tilde{\varepsilon}_2 h^*)] - u(\pi_c^*)\}F'(k/h_2^*)k/h_2^{*2}, \tag{47}$$

where $h_2^* = R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)$ and $\pi_c^* = r_x(x_h^*) + r_y(y_h^*) - c_x(x_h^* + x_f^*) + e_0 h_2^*$. Since $\tilde{\varepsilon}_1$ has a mean of zero, we can write expression (47) as

$$\{u'(\pi_c^*) - \mathrm{E}_2[u'(\pi_c^* + \tilde{\varepsilon}_2 h_2^*)]\}\int_{k/h_2^*}^{\infty}\varepsilon_1\ \mathrm{d}F(\varepsilon_1)$$
$$+\{\mathrm{E}_2[u(\pi_c^* + \tilde{\varepsilon}_2 h_2^*)] - u(\pi_c^*)\}F'(k/h_2^*)k/h_2^{*2}. \tag{48}$$

Since $\mathrm{E}_2(\tilde{\varepsilon}_2) = 0$, it follows from risk aversion and Jensen's inequality that $u(\pi_c^*) > \mathrm{E}_2[u(\pi_c^* + \tilde{\varepsilon}_2 h_2^*)]$ and thus the second term of expression (23) is unambiguously negative. If the MNF is prudent, it follows from $\mathrm{E}_2(\tilde{\varepsilon}_2) = 0$ and Jensen's inequality that $u'(\pi_c^*) \leq \mathrm{E}_2[u'(\pi_c^* + \tilde{\varepsilon}_2 h_2^*)]$, where the equality holds only when $u'''(\pi) \equiv 0$. Since $\tilde{\varepsilon}_1$ has a mean of zero, the first-term of expression (48) is non-positive, thereby implying that expression (48) is also non-positive, should $u'''(\pi) \geq 0$. The following proposition is then an immediate consequence of Eq. (39) and the second-order conditions of program (34).

Proposition 7. *Suppose that the risk-averse MNF has access to the intertemporally unbiased currency futures market for hedging purposes and is liquidity constrained, i.e.,* $0 < k < \infty$. *If the MNF is prudent, i.e.,* $u'''(\pi) \geq 0$, *its optimal nearby futures position,* h_1^*, *satisfies that* $h_1^* < 0$.

The intuition of Proposition 7 is as follows. In the absence of the liquidity constraint, Proposition 1 implies that a full-hedge with $h_1 = 0$ and $h_2 = R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)$ completely eliminates the exchange rate risk. When the liquidity constraint is present, such a full-hedge is no longer optimal for the MNF because it creates liquidity risk gauged by the residual risk, $\tilde{\varepsilon}_2[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$, arising from the premature liquidation of the distant futures position at $t = 1$, as is evident from Eq. (29). If the MNF has a quadratic utility function, it is well-known that its optimal hedge position is the one that minimizes the variability of its random global domestic currency profit at $t = 2$. Note that the residual risk due to the premature liquidation of the distant futures position at $t = 1$ prevails for all $\varepsilon_1 > k/(h_1 + h_2)$. The liquidity constrained MNF as such finds it optimal to set $h_2^* = R_x(x_f^*) + R_y(y_f^*) - c_y(y_h^* + y_f^*)$ to minimize the extent of the exchange rate risk and set $h_1^* < 0$ to limit the degree of liquidity risk by shrinking the interval over which the distant futures position is prematurely liquidated at $t = 1$, when its utility function is quadratic.

If the MNF is prudent, it is more sensitive to low realizations of its random global domestic currency profit at $t = 2$ than to high ones (Kimball, 1990, 1993). Since the low realizations of its random global domestic currency profit at $t = 2$ occur when the distant futures position is prematurely liquidated at $t = 1$ and the realized values of $\tilde{\varepsilon}_1$ are negative, the MNF can avoid these realizations by longing the nearby futures contracts, as is evident from Eqs. (29) and (30). The incentive to long the nearby futures contracts as described under quadratic utility functions is therefore reinforced when the MNF becomes prudent so that $h_1^* < 0$.

Proposition 7 shows that the prudent MNF optimally chooses a long nearby futures position, i.e., $h_1^* < 0$, and a short distant futures position, i.e., $h_2^* > 0$, in the presence of the liquidity constraint. Such a trading strategy constitutes a futures spread that involves a long or short position in one futures contract and an opposite position in another. Since we have assumed away any biasedness and spread basis in the currency futures market, Proposition 7 offers a rationale for the pure hedging demand for futures spreads by prudent MNFs that encounter liquidity constraints (see also Wong, 2004b).

5. Hedging role of options

In this section, we introduce currency call option contracts that expire at $t = 1$ to the MNF.[15] We are particularly interested in scrutinizing the hedging role of options. To this end, we assume that the one-period call option contracts are fairly priced in that the option premium, p is set equal to the expected value of $\max(\tilde{e}_1 - s, 0)$, where s is the strike price of the currency call options. To further simplify the analysis, we set $s = e_0$ so that the one-period currency call options are at the money at $t = 0$.

As in the previous section, the MNF sells (purchases if negative) h_1 and h_2 units of the nearby and distant futures contracts at $t = 0$ at the same initial futures exchange rates, $f_0 = e_0$. The MNF also writes (buys if negative) z units of the one-period, at-the-money, currency call options at $t = 0$. Conditional on the realized value of the spot exchange rate at $t = 1$, the MNF enjoys a net gain (or suffers a net loss if negative) from its hedge position, (h_1, h_2, z), equal to $(f_0 - e_1)(h_1 + h_2) - \max(e_1 - e_0, 0)z$. The MNF is liquidity constrained in that it is obliged to terminate its distant futures position whenever the net loss incurred at $t = 1$ exceeds the prespecified threshold level, $k > 0$. Specifically, if $(e_1 - f_0)(h_1 + h_2) + \max(e_1 - e_0, 0)z > k$, the MNF prematurely liquidates its distant futures position at $t = 1$, implying that its random global domestic currency profit at $t = 2$ is given by

$$\tilde{\pi}_\ell = r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) + \tilde{e}_2[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$$

[15] We do not consider currency put option contracts because they are redundant in that they can be readily replicated by combinations of currency futures and call option contracts (Sercu and Uppal, 1995).

$$+(f_0 - e_1)(h_1 + h_2) + [p - \max(e_1 - e_0, 0)]z$$

$$= r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) + (e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$$

$$-\varepsilon_1(h_1 + h_2) + [p - \max(\varepsilon_1, 0)]z, \quad (49)$$

where the second equality follows from the intertemporal unbiasedness of the currency futures market and the random walk assumption about the spot exchange rate dynamics. On the other hand, if $(e_1 - f_0)(h_1 + h_2) + \max(e_1 - e_0, 0)z \leq k$, the MNF holds its distant futures position until $t = 2$ so that its random global domestic currency profit at $t = 2$ becomes

$$\tilde{\pi}_c = r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) + \tilde{e}_2[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$$

$$+(f_0 - e_1)h_1 + (f_0 - \tilde{e}_2)h_2 + [p - \max(e_1 - e_0, 0)]z$$

$$= r_x(x_h) + r_y(y_h) - c_x(x_h + x_f) + (e_0 + \varepsilon_1 + \tilde{\varepsilon}_2)[R_x(x_f) + R_y(y_f) - c_y(y_h + y_f)]$$

$$-\varepsilon_1(h_1 + h_2) - \tilde{\varepsilon}_2 h_2 + [p - \max(\varepsilon_1, 0)]z, \quad (50)$$

where the second equality follows from the intertemporal unbiasedness of the currency futures market and the random walk assumption about the spot exchange rate dynamics.

From the previous section, we know that the MNF optimally chooses $h_1^* + h_2^* > 0$ when $z = 0$. In this case, the MNF's maximum expected utility is given by

$$EU^* = \int_{-\infty}^{k/(h_1^*+h_2^*+z)} u(\pi_c^*)\ \mathrm{d}F(\varepsilon_1) + \int_{k/(h_1^*+h_2^*+z)}^{\infty} \mathrm{E}_2[u(\tilde{\pi}_\ell)]\ \mathrm{d}F(\varepsilon_1), \quad (51)$$

where $\pi_c^* = r_x(x_h^*) + r_y(y_h^*) - c_x(x_h^* + x_f^*) + e_0 h_2^* - \varepsilon_1 h_1^* + [p - \max(\varepsilon_1, 0)]z$, $\tilde{\pi}_\ell^* = \pi_c^* + \tilde{\varepsilon}_2 h_2^*$, $z = 0$, and the optimal solution is described in Proposition 5. To show the hedging role of the one-period currency call options, we partially differentiate EU^* defined in Eq. (51) with respect to z and evaluate the resulting derivative at $z = 0$ to yield

$$\left.\frac{\partial EU^*}{\partial z}\right|_{z=0} = \int_{-\infty}^{k/(h_1^*+h_2^*)} u'(\pi_c^*)[p - \max(\varepsilon_1, 0)]\ \mathrm{d}F(\varepsilon_1)$$

$$+\int_{k/(h_1^*+h_2^*)}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell)](p-\varepsilon_1)\ \mathrm{d}F(\varepsilon_1)$$

$$+\mathrm{E}_2[u(\tilde{\pi}_{\ell 1}^*)-u(\tilde{\pi}_{c1}^*)]F'[k/(h_1^*+h_2^*)]k/(h_1^*+h_2^*)^2, \tag{52}$$

where we have used Leibniz's rule. Substituting Eq. (39) into the right-hand side of Eq. (52) yields

$$\left.\frac{\partial EU^*}{\partial z}\right|_{z=0} = \int_{-\infty}^{k/(h_1^*+h_2^*)} u'(\pi_c^*)[p-\max(-\varepsilon_1,0)]\ \mathrm{d}F(\varepsilon_1)$$

$$+\int_{k/(h_1^*+h_2^*)}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell)]p\ \mathrm{d}F(\varepsilon_1). \tag{53}$$

If the right-hand side of Eq. (53) is positive (negative), it follows from the strict concavity of the MNF's objective function that the MNF optimally chooses a short (long) call option position, i.e., $z^* > (<)\ 0$. The following proposition shows that the right-hand side of Eq. (53) is unambiguously negative should the MNF be prudent.

Proposition 8. *Suppose that the risk-averse MNF has access to the intertemporally unbiased currency futures market for hedging purposes and is liquidity constrained, i.e., $0 < k < \infty$. If the MNF is prudent and can also use the one-period currency call option contracts that are at the money and fairly priced, the MNF's optimal call option position, z^*, satisfies that $z^* < 0$.*

Proof. Define the following:

$$m = \int_{-\infty}^{k/(h_1^*+h_2^*)} u'(\pi_c^*)\ \mathrm{d}F(\varepsilon_1) + \int_{k/(h_1^*+h_2^*)}^{\infty} \mathrm{E}_2[u'(\tilde{\pi}_\ell)]\ \mathrm{d}F(\varepsilon_1). \tag{54}$$

Using Eq. (54), we can write Eq. (53) as

$$\left.\frac{\partial EU^*}{\partial z}\right|_{z=0} = \int_{-\infty}^{k/(h_1^*+h_2^*)} [u'(\pi_c^*)-m][p-\max(-\varepsilon_1,0)]\ \mathrm{d}F(\varepsilon_1)$$

$$+\int_{k/(h_1^*+h_2^*)}^{\infty} \{\mathrm{E}_2[u'(\tilde{\pi}_\ell)]-m\}p\ \mathrm{d}F(\varepsilon_1). \tag{55}$$

since p equals the expectation of $\max(-\tilde{\varepsilon}_1, 0)$.[16] Inspection of Eqs. (54) and (55) reveals that Eq. (55) can be simplified to

$$\left.\frac{\partial EU^*}{\partial z}\right|_{z=0} = \int_{-\infty}^{0} [u'(\pi_c^*) - m]\varepsilon_1 \,\mathrm{d}F(\varepsilon_1). \tag{56}$$

Since $\tilde{\varepsilon}_2$ has a mean of zero, prudence and Jensen's inequality imply that $u'(\pi_c^*) \leq \mathrm{E}_2[u'(\tilde{\pi}_\ell^*)]$, where the equality holds only when $u'''(\pi) \equiv 0$. From Eq. (54), we have

$$m \geq \int_{-\infty}^{\infty} u'(\pi_c^*) \,\mathrm{d}F(\varepsilon_1) \geq u'[r_x(x_h^*) + r_y(y_h^*) - c_x(x_h^* + x_f^*) + e_0 h_2^*], \tag{57}$$

where the second inequality follows from prudence and the zero-mean of $\tilde{\varepsilon}_1$, and the equality holds only when $u'''(\pi) \equiv 0$. From Proposition 5, we know that $h_1^* < 0$ so that $\partial u'(\pi_c^*)/\partial\varepsilon_1 = -u''(\pi_c^*)h_1^* < 0$. Thus, it follows from inequality (57) that there must exist a unique point, $\hat{\varepsilon}_1 \leq 0$, at which $m = u'(\pi_c^*)$, where the equality holds only when $u'''(\pi) \equiv 0$. Using Eq. (54), we can write Eq. (56) as

$$\left.\frac{\partial EU^*}{\partial z}\right|_{z=0} = \int_{-\infty}^{0} [u'(\pi_c^*) - m](\varepsilon_1 - \hat{\varepsilon}_1) \,\mathrm{d}F(\varepsilon_1) + \int_{-\infty}^{0} [u'(\pi_c^*) - m]\hat{\varepsilon}_1 \,\mathrm{d}F(\varepsilon_1). \tag{58}$$

Since $u'(\pi_c^*) > (<)\ m$ for all $\varepsilon_1 < (>)\ \hat{\varepsilon}_1$, the first term on the right-hand side of Eq. (58) is negative. Define the following:

$$n = \int_{-\infty}^{k/(h_1^*+h_2^*)} u'(\pi_c^*) \,\mathrm{d}F(\varepsilon_1)/F[k/(h_1^* + h_2^*)]. \tag{59}$$

Since both $u'(\pi_c^*)$ and $\mathrm{E}_2[u'(\tilde{\pi}_\ell^*)]$ are decreasing in ε_1, it must be true that $n > m$ as n is the expected value of $u'(\pi_c^*)$ conditional on $\varepsilon_1 \leq k/(h_1^* + h_2^*)$, i.e., the low realizations of $\tilde{\varepsilon}_1$. Using Eq. (59), we have

$$\int_{-\infty}^{0} [u'(\pi_c^*) - m] \,\mathrm{d}F(\varepsilon_1) = F[k/(h_1^* + h_2^*)](n - m)$$
$$+ \int_{0}^{k/(h_1^*+h_2^*)} [m - u'(\pi_c^*)] \,\mathrm{d}F(\varepsilon_1). \tag{60}$$

[16] Since the one-period currency call options are fairly priced, p equals the expectation of $\max(\tilde{\varepsilon}_1, 0)$. It then follows from the zero-mean of $\tilde{\varepsilon}_1$ that p also equals the expectation of $\max(-\tilde{\varepsilon}_1, 0)$.

The first term on the right-hand side of Eq. (60) is positive because $n > m$. The second term is also positive because $m > u'(\pi_c^*)$ for all $\varepsilon_1 > \hat{\varepsilon}_1$ and $\hat{\varepsilon}_1 \leq 0$. Hence, the second term on the right-hand side of Eq. (58) is non-positive. This completes our proof. □

The intuition of Proposition 8 is as follows. Suppose that the MNF does not use the one-period currency call options for hedging purposes, i.e., $z = 0$. In this case, we know from Proposition 5 that the MNF faces no exchange rate risk only when its distant futures position, h_2^*, is continued at $t = 1$, which occurs for all $\varepsilon_1 \leq k/(h_1^* + h_2^*)$. If the MNF has a quadratic utility function, it is well-known that its optimal hedge position is the one that minimizes the variability of its random global domestic currency profit at $t = 2$. To further improve the hedging performance, the MNF finds it optimal to opt for $z < 0$ so as to make the distant futures position continue for all $\varepsilon_1 \leq k/(h_1^* + h_2^* + z)$. The liquidity constrained MNF as such uses the currency call options as a hedging instrument when its utility function is quadratic.

Given prudence, the MNF is more sensitive to low realizations of its random global domestic currency profit at $t = 2$ than to high ones (Kimball, 1990, 1993). Since the low realizations of its random global domestic currency profit at $t = 2$ occur when the distant futures position is prematurely liquidated at $t = 1$ and the realized values of $\tilde{\varepsilon}_1$ are negative, the MNF can avoid these realizations by buying the currency call option contracts, as is evident from Eqs. (49) and (50). The incentive to purchase the currency call options as described under quadratic utility functions is thus reinforced when the MNF becomes prudent so that $z^* < 0$.

Options are particularly useful for prudent MNFs facing liquidity constraints because of their asymmetric payoff profiles, vis-à-vis the symmetric payoff profiles of futures. In the 1998 Wharton survey of financial risk management by US non-financial firms, Bodnar, Hayt, and Marston (1998) report that 68% of the 200 derivatives-using firms indicated that they had used some form of options within the past 12 months. In light of Proposition 8, the prevalence of liquidity constraints is likely to account for the hedging demand for currency options by prudent MNFs (see also Wong and Xu, 2006; Meng and Wong, 2007).

6. Conclusions

In this paper, we have examined the impact of liquidity risk on the behavior of the risk-averse multinational firm (MNF) under exchange rate uncertainty in a two-period dynamic setting. The MNF has operations domiciled in the home country and in a foreign country, each of which produces a single homogeneous good to be sold in the home and foreign markets. To hedge the exchange rate risk, the MNF is restricted to use currency futures contracts as the sole hedging instrument. The currency futures contracts are marked to market in that they require interim cash settlement of gains and losses at the end of the first period. We have introduced liquidity risk to the MNF by a liquidity constraint that obliges the MNF to prematurely liquidate its futures position whenever the interim loss incurred from this position exceeds a prespecified threshold level.

We have shown that the liquidity constrained MNF optimally opts for an under-hedge should it be prudent. Under-hedging is called for to strike a balance between the extent of the exchange rate risk and that of the liquidity risk. Furthermore, we have shown that the liquidity constrained MNF demands a positive risk premium on its foreign sales, which creates a wedge between the marginal revenues in the home and foreign markets. In response to the imposition of the liquidity constraint, the MNF optimally sells less (more) and produces more (less) in the foreign (home) country. The liquidity constrained MNF as such receives a lower expected global domestic currency profit and attains a lower expected utility level than in the case when the liquidity constraint is absent.

Finally, we have expanded the set of hedging instruments made available to the MNF to include nearby currency futures and option contracts, both of which mature at the end of the first period. We have shown that the liquidity constrained MNF optimally opts for a long nearby futures position and a short distant futures position if it is prudent. Such a trading strategy constitutes a futures spread that involves a long or short position in one futures contract and an opposite position in another. We have shown further that the liquidity constrained MNF optimally uses the one-period currency option contracts for hedging purposes in general, and opts for a long option position if it is prudent in particular. The prevalence of liquidity constraints thus offers a rationale for the optimality of using futures spreads and currency options by prudent MNFs.

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