Liquidity Risk and Corporate Hedging with Futures *

Maurice K. S. Tse, Kit Pong Wong †

School of Economics and Finance
University of Hong Kong

March 2008

This paper examines the hedging behavior of a value-maximizing firm that lasts for two periods. The firm faces uncertain income and is subject to tax asymmetries with no loss-offset provisions. The firm has access to unbiased futures contracts in each period for hedging purposes. We impose a liquidity constraint on the firm. Specifically, whenever the net interim loss due to its first-period futures position exceeds a predetermined threshold level, the firm is forced to terminate its risk management program and thus is prohibited from trading the futures contracts in the second period. We show that the liquidity constrained firm optimally adopts a full-hedge via its second-period futures position to minimize the extent of the income risk, and an under-hedge via its first-period futures position to limit the degree of the liquidity risk.

JEL classification: D21; D81

Keywords: Liquidity risk; Tax asymmetries; Futures; Multi-period hedging

---

*We gratefully acknowledge financial support from a grant provided by the University Grants Committee of the Hong Kong Special Administrative Region, China (Project No. AoE/H-05/99).

† *Correspondence to:* Kit Pong Wong, School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong. Tel.: (852) 2859-1044, fax: (852) 2548-1152, e-mail: kpwong@econ.hku.hk (K. P. Wong).

Liquidity Risk and Corporate Hedging with Futures

This paper examines the hedging behavior of a value-maximizing firm that lasts for two periods. The firm faces uncertain income and is subject to tax asymmetries with no loss-offset provisions. The firm has access to unbiased futures contracts in each period for hedging purposes. We impose a liquidity constraint on the firm. Specifically, whenever the net interim loss due to its first-period futures position exceeds a predetermined threshold level, the firm is forced to terminate its risk management program and thus is prohibited from trading the futures contracts in the second period. We show that the liquidity constrained firm optimally adopts a full-hedge via its second-period futures position to minimize the extent of the income risk, and an under-hedge via its first-period futures position to limit the degree of the liquidity risk.

JEL classification: D21; D81

Keywords: Liquidity risk; Tax asymmetries; Futures; Multi-period hedging

1. Introduction

Liquidity risk can be classified either as asset liquidity risk or as funding liquidity risk (Jorion, 2001). The former refers to the risk that the liquidation value of the assets differs significantly from the prevailing mark-to-market value. The latter refers to the risk that payment obligations cannot be met due to inability to raise new funds. Liquidity risk can be fatal to a firm even when the firm is technically solvent. Prominent examples include the case of Metallgesellschaft and the debacle of Long-Term Capital Management.

The Committee on Payments and Settlement Systems (1998) regards liquidity risk as one of the risks that users of derivatives and other financial contracts must take into account when devising their risk management strategies. In light of this advice, the purpose of this paper is thus to examine the effect of funding liquidity risk on corporate risk management. To this end, we develop a two-period model of a value-maximizing firm that is entitled to receive uncertain income at the end of the second period. The firm has incentives to hedge because its uncertain income is subject to asymmetric taxation with no loss-offset provisions (Smith and Stulz, 1985).

The firm has access to unbiased futures contracts in each period for hedging purposes. There are ex-post funding needs due to the interim cash settlement of gains and losses from futures positions in the first period. We follow Lien (2003) and Wong (2004, 2005) to impose a liquidity contract on the firm. Specifically, whenever the net interim loss due to its first-period futures position exceeds a predetermined threshold level, we assume that the firm is forced to terminate its risk management program. The firm as such is prohibited from trading the futures contracts in the second period. The presence of the liquidity constraint gives rise to liquidity risk in addition to the income risk already faced by the firm.

If the liquidity constraint is absent, it is well known that the firm optimally adopts a full-hedge in each period. Doing so completely eliminates the income risk (see Pirrong, 1993; Lien, 1999; and Wong, 2005). This full-hedging result follows from the fact that the firm's tax liability can be viewed as an option on the before-tax profit granted by the firm to the government. Since the value of any option is positively related to the riskiness of the underlying asset (Merton, 1973), the firm finds it optimal to drive the option value of its tax liability down to zero by eliminating all the income risk.

In the presence of the liquidity constraint, it is not possible to completely eliminate the income risk due to the termination of the risk management program when the interim loss from the first-period futures position reaches an alarming level. We show that the firm's income risk exposure has a short-term and a long-term components. To hedge against the long-term component, the firm optimally opts for a full-hedge via its second-period futures position. The short-term component, which is made complicated by the liquidity risk, is better hedged by adopting an under-hedge via the first-period futures position. The value-maximizing firm's incentive to hedge is adversely affected when the liquidity constraint prevails.

The rest of this paper is organized as follows. Section 2 delineates a two-period model of a value-maximizing firm facing uncertain income that is subject to asymmetric taxation with no loss-offset provisions. The firm has access to unbiased futures contracts in each period for hedging purposes. A liquidity constraint is imposed on the firm. Section 3 derives the optimal futures hedging decisions. The final section concludes.

2. The model

Consider a firm that makes decisions in a two-period horizon with three dates (indexed by $t = 0$, 1, and 2). The firm is risk neutral and value maximizing. Interest rates in both periods are known at $t = 0$ with certainty. To simplify notation, we henceforth suppress the interest factors by compounding all cash flows to their future values at $t = 2$.

The firm initially looks forward to receiving ordinary income of $I + y$ at $t = 2$, where I denotes the deterministic component and y denotes the stochastic component. We assume that y is distributed over the real line with mean zero. To hedge against the income risk, y, the firm can trade infinitely divisible futures contracts at the beginning of each period. All futures contracts transacted at date t expire at date $t + 1$, where $t = 0$ and 1.

Denote f_t as the futures price at date t, where $t = 0$, 1, and 2. We assume that there is no basis risk so that f_2 is set exactly equal to y. While f_0 is known at $t = 0$, f_1 and f_2 are regarded as random variables distributed over the real line *ex ante*. Let h_1 and h_2 be the long (short if negative) first-period and second-period futures positions established by the firm at $t = 0$ and $t = 1$, respectively.

To focus on the firm's hedging motive, *vis-à-vis* its speculative motive, we assume that the futures contracts in each period are unbiased. To wit, we set $f_0 = 0$ so that f_0 is equal to the unconditional expected value of y. Conditioned on the realization of f_1, we specify $y = f_1 + \epsilon$, where ϵ is a zero-mean random variable conditionally independent of f_1, thereby rendering f_1 to be the conditional expectation of y. By the law of iterated expectations, it follows from the zero unconditional mean of y that f_1 is a zero-mean random variable.

At $t = 1$, the firm enjoys a gain (or suffers a loss if negative) of $(f_1 - f_0)h_1 = f_1 h_1$ from its first-period futures position, h_1. Following Lien (2003) and Wong (2004, 2005), the firm is liquidity constrained in that it is forced to terminate its risk management program whenever the loss incurred at $t = 1$ exceeds a predetermined threshold level, k, i.e., whenever $-f_1 h_1 > k$. In the presence of the liquidity constraint, the firm's before-tax profit at $t = 2$ is therefore given by

$$\pi = \begin{cases} I + f_1 + \epsilon + f_1 h_1 + \epsilon h_2 & \text{if } -f_1 h_1 \le k, \\ I + f_1 + \epsilon + f_1 h_1 & \text{if } -f_1 h_1 > k, \end{cases}$$

where $(f_1 + \epsilon - f_1)h_2 = \epsilon h_2$ is the gain/loss from the second-period futures position, h_2. The before-tax profit at $t = 2$ is subject to a constant tax rate, τ, with $0 < \tau < 1$. The tax system is asymmetric with no loss-offset provisions. That is, corporate income taxes are paid only when before-tax profits are positive.

Anticipating the impact of the liquidity constraint at $t = 1$, the firm evaluates at $t = 0$ that its expected after-tax profit at $t = 2$ is given by

$$\Pi = I - \tau \int_{-\infty}^{-\frac{k}{h_1}} \int_{-\infty}^{\infty} \max[I + f_1 + \epsilon + f_1 h_1, 0]\, \mathrm{d}\Phi(\epsilon)\, \mathrm{d}\Psi(f_1)$$

$$-\tau \int_{-\frac{k}{h_1}}^{\infty} \int_{-\infty}^{\infty} \max[I + f_1 + \epsilon + f_1 h_1 + \epsilon h_2, 0]\, \mathrm{d}\Phi(\epsilon)\, \mathrm{d}\Psi(f_1) \tag{1}$$

if $h_1 > 0$,

$$\Pi = I - \tau \int_{-\infty}^{-\infty} \int_{-\infty}^{\infty} \max[I + f_1 + \epsilon + \epsilon h_2, 0]\, \mathrm{d}\Phi(\epsilon)\, \mathrm{d}\Psi(f_1) \tag{2}$$

if $h_1 = 0$, and

$$\Pi = I - \tau \int_{-\infty}^{-\frac{k}{h_1}} \int_{-\infty}^{\infty} \max[I + f_1 + \epsilon + f_1 h_1 + \epsilon h_2, 0]\, \mathrm{d}\Phi(\epsilon)\, \mathrm{d}\Psi(f_1)$$

$$-\tau \int_{-\frac{k}{h_1}}^{\infty} \int_{-\infty}^{\infty} \max[I + f_1 + \epsilon + f_1 h_1, 0]\, \mathrm{d}\Phi(\epsilon)\, \mathrm{d}\Psi(f_1) \tag{3}$$

if $h_1 < 0$, where $\Psi(f_1)$ and $\Phi(\epsilon)$ are the cumulative distribution functions of f_1 and ϵ, respectively.

3. Optimal futures hedging decisions

As usual, we solve the firm's multi-period decision problems by using backward induction. At $t = 1$, given that $-f_1h_1 \leq k$, the firm is allowed to continue its risk management program in the second period. The firm as such chooses its second-period futures position, h_2, so as to maximize its expected after-tax profit at $t = 2$:

$$I + f_1 + f_1h_1 - \tau \int_{-\infty}^{\infty} \max[I + f_1 + \epsilon + f_1h_1 + \epsilon h_2, 0]\ \mathrm{d}\Phi(\epsilon), \tag{4}$$

conditioned on the realized value of f_1 and on the fact that $-f_1h_1 \leq k$.

Since the maximum operator, $\max[\xi, 0]$, is convex in its argument, ξ, Jensen's inequality implies that

$$\int_{-\infty}^{\infty} \max[I + f_1 + \epsilon + f_1h_1 + \epsilon h_2, 0]\ \mathrm{d}\Phi(\epsilon) \geq \max[I + f_1 + f_1h_1, 0],$$

where the equality holds only when $h_2 = -1$. To minimize its expected tax liability in equation (4), the firm finds it optimal to opt for a full-hedge via its second-period futures position, i.e., $h_2 = -1$, thereby invoking the following proposition.

Proposition 1. *Given that the futures contracts in the second period are unbiased, the liquidity constrained firm optimally adopts a full-hedge via its second-period futures position, i.e.,* $h_2 = -1$, *when* $-f_1h_1 \leq k$ *at* $t = 1$.

The intuition of Proposition 1 is as follows. When $-f_1h_1 \leq k$ at $t = 1$, the firm is allowed to continue its risk management program in the second period. In this case, the firm can adopt a full-hedge via its second-period futures position so as to completely eliminate the income risk, $y = f_1 + \epsilon$, which affects both the before-tax profit and the tax liability of the firm at $t = 2$. Doing so is optimal because the firm's tax liability can be viewed as an option on the firm's before-tax profit granted by the firm to the government. Since the value of any option is positively related to the riskiness of the underlying asset (Merton, 1973), the firm finds it optimal to reduce the option value of its tax liability as much as possible. This option value can be set equal to zero by selecting $h_2 = -1$ at $t = 1$ (see also Pirrong, 1995; Lien, 1999; and Wong, 2005).

Now, we go back to the first period. In order to derive the optimal first-period futures position, h_1, we need to know which equation, (1), (2), or (3), contains the solution. To this end, we substitute $h_2 = -1$ into equation (1) to get

$$\Pi = I - \tau \int_{-\infty}^{-\frac{k}{h_1}} \int_{-I-f_1-f_1h_1}^{\infty} (I + f_1 + \epsilon + f_1h_1)\ \mathrm{d}\Phi(\epsilon)\ \mathrm{d}\Psi(f_1)$$

$$-\tau \int_{\max[-\frac{k}{h_1}, -\frac{I}{1+h_1}]}^{\infty} (I + f_1 + f_1h_1)\ \mathrm{d}\Psi(f_1), \tag{5}$$

if $h_1 > 0$. Likewise, we substitute $h_2 = -1$ into equation (3) to get

$$\Pi = I - \tau \int_{-\frac{I}{1+h_1}}^{\max[-\frac{k}{h_1}, -\frac{I}{1+h_1}]} (I + f_1 + f_1h_1)\ \mathrm{d}\Psi(f_1)$$

$$-\tau \int_{-\frac{k}{h_1}}^{\infty} \int_{-I-f_1-f_1h_1}^{\infty} (I + f_1 + \epsilon + f_1h_1)\ \mathrm{d}\Phi(\epsilon)\ \mathrm{d}\Psi(f_1) \tag{6}$$

if $-1 < h_1 < 0$, and

$$\Pi = I - \tau \int_{-\infty}^{\min[-\frac{k}{h_1}, -\frac{I}{1+h_1}]} (I + f_1 + f_1h_1)\ \mathrm{d}\Psi(f_1)$$

$$-\tau \int_{-\frac{k}{h_1}}^{\infty} \int_{-I-f_1-f_1h_1}^{\infty} (I + f_1 + \epsilon + f_1h_1)\ \mathrm{d}\Phi(\epsilon)\ \mathrm{d}\Psi(f_1). \tag{7}$$

if $h_1 < -1$.

Using Leibniz's rule to partially differentiate Π as defined in equation (5) with respect to h_1 for h_1 sufficiently close to zero yields

$$\frac{\partial \Pi}{\partial h_1} = -\tau \int_{-\infty}^{-\frac{k}{h_1}} \int_{-I-f_1-f_1 h_1}^{\infty} f_1 \, \mathrm{d}\Phi(\epsilon) \, \mathrm{d}\Psi(f_1)$$

$$-\tau \int_{-I+\frac{k(1+h_1)}{h_1}}^{\infty} \left[I - \frac{k(1+h_1)}{h_1} + \epsilon\right] \mathrm{d}\Phi(\epsilon) \; \psi\Big(-\frac{k}{h_1}\Big)\frac{k}{h_1^2}$$

$$-\tau \int_{-\frac{I}{1+h_1}}^{\infty} f_1 \, \mathrm{d}\Psi(f_1). \tag{8}$$

Using Leibniz's rule to partially differentiate Π as defined in equation (6) with respect to h_1 for h_1 sufficiently close to zero yields

$$\frac{\partial \Pi}{\partial h_1} = -\tau \int_{-\frac{I}{1+h_1}}^{-\frac{k}{h_1}} f_1 \, \mathrm{d}\Psi(f_1) - \tau\Big[I - \frac{k(1+h_1)}{h_1}\Big]\psi\Big(-\frac{k}{h_1}\Big)\frac{k}{h_1^2}$$

$$-\tau \int_{-\frac{k}{h_1}}^{\infty} \int_{-I-f_1-f_1 h_1}^{\infty} f_1 \, \mathrm{d}\Phi(\epsilon) \, \mathrm{d}\Psi(f_1)$$

$$+\tau \int_{-I+\frac{k(1+h_1)}{h_1}}^{\infty} \left[I - \frac{k(1+h_1)}{h_1} + \epsilon\right] \mathrm{d}\Phi(\epsilon) \; \psi\Big(-\frac{k}{h_1}\Big)\frac{k}{h_1^2}. \tag{9}$$

Evaluating equation (8) at $h_1 \to 0^+$ and equation (9) at $h_1 \to 0^-$ yields

$$\frac{\partial \Pi}{\partial h_1}\bigg|_{h_1=0} = \lim_{h_1 \to 0^+} \frac{\partial \Pi}{\partial h_1} = \lim_{h_1 \to 0^-} \frac{\partial \Pi}{\partial h_1} = -\tau \int_{-I}^{\infty} f_1 \, \mathrm{d}\Psi(f_1) < 0. \tag{10}$$

From the strict concavity of Π as defined in equations (1) and (3), equation (10) implies that the firm optimally chooses $h_1 < 0$ and thus the solution must be contained in equation (3).

Using Leibniz's rule to partially differentiate Π as defined in equation (6) with respect to h_1 for h_1 sufficiently close to -1 yields

$$\frac{\partial \Pi}{\partial h_1} = -\tau \int_{-\frac{I}{1+h_1}}^{-\frac{k}{h_1}} f_1 \, \mathrm{d}\Psi(f_1) - \tau\Big[I - \frac{k(1+h_1)}{h_1}\Big]\psi\Big(-\frac{k}{h_1}\Big)\frac{k}{h_1^2}$$

$$-\tau \int_{-\frac{k}{h_1}}^{\infty} \int_{-I-f_1-f_1 h_1}^{\infty} f_1 \, \mathrm{d}\Phi(\epsilon) \, \mathrm{d}\Psi(f_1)$$

$$+\tau \int_{-I+\frac{k(1+h_1)}{h_1}}^{\infty} \left[I - \frac{k(1+h_1)}{h_1} + \epsilon \right] \mathrm{d}\Phi(\epsilon) \, \psi\Big(-\frac{k}{h_1} \Big) \frac{k}{h_1^2}. \tag{11}$$

Using Leibniz's rule to partially differentiate Π as defined in equation (7) with respect to h_1 for h_1 sufficiently close to -1 yields

$$\frac{\partial \Pi}{\partial h_1} = -\tau \int_{-\infty}^{-\frac{k}{h_1}} f_1 \, \mathrm{d}\Psi(f_1) - \tau \Big[I - \frac{k(1+h_1)}{h_1} \Big] \psi\Big(-\frac{k}{h_1} \Big) \frac{k}{h_1^2}$$

$$-\tau \int_{-\frac{k}{h_1}}^{\infty} \int_{-I-f_1-f_1 h_1}^{\infty} f_1 \, \mathrm{d}\Phi(\epsilon) \, \mathrm{d}\Psi(f_1)$$

$$+\tau \int_{-I+\frac{k(1+h_1)}{h_1}}^{\infty} \left[I - \frac{k(1+h_1)}{h_1} + \epsilon \right] \mathrm{d}\Phi(\epsilon) \, \psi\Big(-\frac{k}{h_1} \Big) \frac{k}{h_1^2}. \tag{12}$$

Evaluating equation (11) at $h_1 \to -1^+$ and equation (12) at $h_1 \to -1^-$ yields

$$\frac{\partial \Pi}{\partial h_1}\Big|_{h_1=-1} = \lim_{h_1 \to -1^+} \frac{\partial \Pi}{\partial h_1} = \lim_{h_1 \to -1^-} \frac{\partial \Pi}{\partial h_1}$$

$$= -\tau \int_{-\infty}^{k} f_1 \, \mathrm{d}\Psi(f_1) - \tau I \psi(k) k - \tau[1 - \Phi(-I)] \int_{k}^{\infty} f_1 \, \mathrm{d}\Psi(f_1)$$

$$+\tau \int_{-I}^{\infty} (I + \epsilon) \, \mathrm{d}\Phi(\epsilon) \, \psi(k) k$$

$$= \tau \Phi(-I) \int_{k}^{\infty} f_1 \, \mathrm{d}\Psi(f_1) - \tau \int_{-\infty}^{-I} (I + \epsilon) \, \mathrm{d}\Phi(\epsilon) \, \psi(k) k > 0. \tag{13}$$

From the strict concavity of Π as defined in equation (3), equation (13) implies that the firm optimally chooses $h_1 > -1$. Therefore, we establish the following proposition.

Proposition 2. *Given that the futures contracts in each period are unbiased, the liquidity constrained firm optimally adopts an under-hedge via its first-period futures position, i.e.,* $-1 < h_1 < 0$, *at* $t = 0$.

To see the intuition of Proposition 2, suppose that the firm adopts a full-hedge via its first-period futures position, i.e., $h_1 = -1$. Then, substituting $h_1 = h_2 = -1$ into equation (3) yields

$$\Pi = (1-\tau)I - \tau[1-\Psi(k)]\int_{-\infty}^{\infty} \max[-(I+\epsilon), 0]\ \mathrm{d}\Phi(\epsilon), \tag{14}$$

where we have used the fact that $I + \epsilon = \max[I + \epsilon, 0] - \max[-(I + \epsilon), 0]$. Inspection of equation (14) reveals that the option value of the firm's tax liability paid by the firm to the government is positive. Succinctly, such an option has the same positive value, τ times the expected value of $\max[-(I+\epsilon), 0]$ with respect to $\Phi(\epsilon)$, whenever the firm is forced to terminate its risk management program at $t = 1$, which occurs with probability $1 - \Psi(k)$. The firm as such has incentives to reduce this probability of paying the option value to the government by reducing its short first-period futures position, as is evident from equation (13). Doing so shrinks the interval, $(-k/h_1, \infty)$, over which the firm is forced to terminate its risk management program at $t = 1$. Thus, this makes an under-hedge in the first period optimal for the liquidity constrained firm.

4. Conclusions

This paper has examined the hedging behavior of a value-maximizing firm that lasts for two periods. The firm faces uncertain income and is subject to asymmetric taxation with no loss-offset provisions. The firm has access to unbiased futures contracts in each period for hedging purposes. As in Lien (2003) and Wong (2004, 2005), we have imposed a liquidity constraint on the firm. Specifically, whenever the interim loss from its first-period futures position exceeds a predetermined threshold level, the firm is forced to terminate its risk management program and thus is prohibited from trading the futures contracts in the second period. This interim funding requirement creates liquidity risk in addition to the income risk already faced by the firm.

Within this context, we have shown that the liquidity constrained firm optimally adopts a full-hedge via its second-period futures position to minimize the extent of the income risk, and an under-hedge via its first-period futures position to limit the degree of the liquidity risk. The presence of the liquidity constraint as such bestows a perverse incentive on the value-maximizing firm regarding its hedging behavior.

References

Committee on Payment and Settlement Systems (1998). OTC derivatives: Settlement procedures and counterparty risk management. Basle, Switzerland: Bank for International Settlements.

Jorion, P. (2001). *Value at Risk: The New Benchmark for Managing Financial Risk* (2nd ed.). New York, NY: McGraw-Hill.

Lien, D. (1999). Uncertain tax rules and futures hedging. *Managerial and Decision Economics* 20, 429–436.

Lien, D. (2003). The effect of liquidity constraints on futures hedging. *Journal of Futures Markets* 23, 603–613.

Merton, R. C. (1973). Theory of rational option pricing. *Bell Journal of Economics and Management Science* 4, 141–183.

Pirrong, C. (1995). The welfare costs of Arkansas Best: The inefficiency of asymmetric taxation of hedging gains and losses. *Journal of Futures Markets* 15, 111–129.

Smith, C. W., Stulz, R. M. (1985). The determinants of firms' hedging policies. *Journal of Financial and Quantitative Analysis* 20, 391–405.

Wong, K. P. (2004). Hedging, liquidity, and the competitive firm under price uncertainty. *Journal of Futures Markets* 24, 697–706.

Wong, K. P. (2005). Tax asymmetry and futures hedging under liquidity constraints. *Managerial and Decision Economics* 26, 271–281.