Estimating the Natural Rate of Unemployment in Hong Kong

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Abstract

This paper uses unobserved components analysis to estimate the natural rate of unemployment in Hong Kong. We assume that the natural rate follows a random walk and is a determinant of the Beveridge curve. We find that the natural rate has increased since the 1990s but nevertheless lies clearly below the actual rate of unemployment. This indicates that unemployment is likely to decrease in the near future.

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1 Introduction

The Hong Kong economy has in recent years been exposed to a series of large negative shocks. In 1998, the Asian crisis caused a decline of around five percent in real GDP. While the economy subsequently strengthened considerably in 1999 and 2000, the first recovery came to an end with the collapse of the global IT bubble and the terrorist attacks of September 11, 2001. Real GDP declined by one percent in the last quarter of 2001, but rebounded in 2002. In the spring of 2003, however, SARS stopped a second recovery, causing another fall in real GDP of roughly half a percent. At the time of writing, the economy once more seems to rebound.

Figure 1: Time plot of $u_t$ and $v_t$

![Figure 1: Time plot of $u_t$ and $v_t$](image)

Note: Sample period 1981:4 to 2003:3

Figure 1 shows the rate of unemployment and the vacancy rate over the period 1981:4 to 2003:3.\(^1\) Unemployment increased over much of the 1990s and up to 2003, while the vacancy rate declined. In 2003 the vacancy rate was roughly two percentage points below

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\(^1\)The data are from the database of the Census and Statistics Department and have been seasonally adjusted.
its level in 1981, whereas unemployment had risen by four percent. This observation suggests that the natural rate of unemployment – the rate of unemployment which would be observed if the economy were in equilibrium – might have increased. If so, we would not expect unemployment to fall in the coming years to levels as low as in the 1980s in spite of the economic recovery. The aim of this paper is to provide an estimate of the natural rate of unemployment.

Our study is closely related to the Groenewold and Tang [3] who use a structural vector autoregressive model of the rate of unemployment and real output to estimate over the period 1982 to 2000 natural unemployment in Hong Kong (as well as in Korea, Singapore and Taiwan). They focus on the impact of the Asian crisis and present an estimate of the natural rate of unemployment that increases only slightly in 1998 and thus suggests that the increase in unemployment at the time mainly was due to transitory factors.

Besides using a longer data sample, our paper complements and extends this analysis in two important ways. First, while Groenewold and Tang provide estimates of the natural rate of unemployment using an non-structural time series approach, the results presented here rely on the Beveridge curve for the theoretical framework. The fact that two such different approaches yield similar estimates of the evolution over time of the natural rate of unemployment in Hong Kong provides some grounds for believing the results. Second, our estimation technique allows us to quantify the uncertainty attached to the estimate of the natural rate of unemployment. We find that the confidence bands are rather narrow, which is further evidence suggesting that our estimate of the natural rate is reliable.

The rest of the paper is structured as follows. Section 2 discusses the concept of the Beveridge curve. Section 3 shows how to use this theoretical framework to estimate the natural rate of unemployment. Since the natural rate is a variable which is not measured directly, we apply the unobserved components estimation technique. Section 4 presents the path of natural unemployment and documents that the estimate seems reasonable in that it has high explanatory power in a simple Phillips curve and an equation for wage inflation in Hong Kong. Section 5 concludes.
2 The Beveridge curve

The relationship between unemployment and vacancies is commonly studied by means of the Beveridge curve.\(^2\) This curve relates the unemployment rate \(u_t\) to the vacancy rate \(v_t\). Figure 2 shows the scatter plot for Hong Kong. The data points describe a hyperbola, so that the functional form

\[
u_t v_t^\alpha = c_t
\]

(1)
captures the link between \(u_t\) and \(v_t\) rather well (\(\alpha = 1\) would give rise to a rectangular hyperbola; the larger \(\alpha\), the more curvature has the Beveridge curve). The variable \(c_t\) determines how far the hyperbola is from the origin. As illustrated in Figure 3, the higher \(c_t\), the larger is \(u_t\) for any given \(v_t\).

Figure 2: Scatter plot of \(u_t\) and \(v_t\)

Note: Sample period 1981:4 to 2003:3

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\(^2\)For an introduction on the Beveridge curve see Blanchard and Diamond [1]. See Shimer [5] for a recent discussion.
The natural rate of employment $u^*_t$ is that level of unemployment which results if the vacancy rate is at its mean, which we denote by $v^*$. Using equation (1), we define

$$u^*_t \equiv \frac{c_t}{(v^*)^\alpha}.$$ 

If we divide both sides of equation (1) by $(v^*)^\alpha$, the Beveridge curve can be written as

$$u_t \left( \frac{v_t}{v^*} \right)^\alpha = u^*_t$$

or alternatively

$$\frac{u_t \left( \frac{v_t}{v^*} \right)^\alpha}{u^*_t} = 1.$$ 

Taking logarithms we obtain

$$\ln \left[ u_t \left( \frac{v_t}{v^*} \right)^\alpha \right] - \ln(u^*_t) = 0. \quad (2)$$

Expression (2) can be thought of as an equilibrium condition that holds on average over time but which can be disturbed by temporary shocks. We define deviations from equilibrium as

$$x_t = \ln \left[ u_t \left( \frac{v_t}{v^*} \right)^\alpha \right] - \ln(u^*_t) + e_t^* \quad (3)$$

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and assume that $e_i^u$ is white noise.

3 Estimation

We use the time series process of the actual rate of unemployment to estimate the path of the natural rate. In particular, we assume that the rate of unemployment adjusts so as to offset past deviations from the Beveridge curve. Allowing, moreover, for autocorrelation in the changes of unemployment, we consider

$$\Delta \ln(u_t) = \beta \Delta \ln(u_{t-1}) - \gamma x_{t-1} + e_t^u, \quad (4)$$

where $e_t^u$ is white noise.$^3$ Equation (4) states that the rate of unemployment tends to rise when it increased last period and if $x_{t-1}$ was low. Note that we use $\ln(u_t)$ rather than $u_t$ to account for the fact that the unemployment rate cannot turn negative. Replacing $x_{t-1}$ with equation (3), we then estimate

$$\Delta \ln(u_t) = \beta \Delta \ln(u_{t-1}) - \gamma \ln\left(\frac{u_{t-1}}{u^{*}_{t-1}}\right) - \ln(u^{*}_{t-1}) + e_t, \quad (5)$$

where $e_t = e_t^u - \gamma e_t^x$ is distributed as $N(0, \sigma_e^2)$. Equation (5) thus implies that unemployment tends to decrease if either $u_{t-1}$ or $v_{t-1}$ were high relative to the natural rate of unemployment. In terms of Kalman filtering, equation (5) constitutes an observation equation.$^4$

The state equation is given by

$$\ln(u^*_t) = \ln(u^*_{t-1}) + e_t^*, \quad (6)$$

where $e_t^* \sim N(0, \sigma^2_e)$. Thus, we assume that the logarithm of the natural rate of unemployment follows a random walk. We estimate the parameters of equations (5) and (6) using Kalman filtering. Table 1 shows the regression output.

We find that $\alpha$ is below unity, which implies that the Beveridge curve in Hong Kong appears to have less curvature than a rectangular hyperbola.$^5$ The AR coefficient $\beta$ is

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$^3$We exclude the constant since it was insignificant in preliminary estimations.

$^4$For a discussion of Kalman filtering, see e.g. Harvey [4].

$^5$However, a Wald test does not reject the hypothesis that $\alpha$ equals unity ($p$-value = 0.17).
Table 1: Unemployment model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.728**</td>
<td>(0.201)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.229*</td>
<td>(0.102)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.403**</td>
<td>(0.123)</td>
</tr>
<tr>
<td>$\sigma^2_e$</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_z$</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>50.328</td>
<td></td>
</tr>
</tbody>
</table>

Note: Maximum likelihood estimates, 1982:2-2003:3. Standard errors in parentheses. */** denotes significance at the five / one percent level.

estimated as 0.23 and the adjustment coefficient $\gamma$ as 0.40. Correspondingly, the median lag, which is given by $\ln(0.5)/\ln(1 - \gamma)$, equals 1.34. This indicates that, if condition (2) was not met in the last period, the unemployment rate moves such that half of the misalignment is corrected after roughly four months. The estimate of the natural rate of unemployment at the end of the sample is 2.99%. With 95% probability, $u_t^*$ lay in the third quarter of 2003 between 2.11% and 4.25%.

Figure 4 shows the estimate of the Beveridge curve at the end of the sample period. The actual data for 2003:3 lie within the 95% confidence interval. However, the vacancy rate of 0.59% is much below its equilibrium value, which is estimated to be 1.90%, while $u_t$ is with 8.30% above the estimate of $u_t^* = 2.99%$.

4 The natural rate of unemployment

Next we construct an estimate of the natural rate of unemployment. Figure 5 shows the smoothed $u_t^*$ together with the actual rate of unemployment. Natural unemployment is estimated as around 3.5 percent in the first half of the 1980s. It then declines to two
percent at the beginning of the 1990s. Thereafter, it rises over time and approaches three percent in 2003.

Actual unemployment was slightly above trend from 1983 to 1986, but fell below the estimated natural rate in 1987.\textsuperscript{6} During the economic expansion at the beginning of the 1990s, \( u_t \) remained significantly below \( u_t^* \), but rose sharply during the Asian crisis. From then onwards, unemployment in Hong Kong seems to have clearly exceeded its natural rate. It thus appears likely that in the absence of further negative shocks, unemployment in Hong Kong will return to a level of around three percent in the near future.

As noted in the introduction, our estimate of the natural rate is closely related to that presented in Groenewold and Tang [3]. Their estimate also suggests that unemployment was above its natural level during most of the 1980s, below it from the end of the 1980s

\textsuperscript{6}For a detailed discussion of economic developments in Hong Kong over the period 1982 to 2000, the interested reader is referred to Groenewold and Tang [3].
to the onset of the Asian crisis and again above it from 1998 onwards. In contrast to our estimate, the path of $u^*_t$ in their paper displays slightly more time variation and is a little higher throughout the sample.

As a last step of the analysis, we study the plausibility of the estimated natural rate of unemployment. In particular, we assess whether the deviation of actual unemployment from its natural rate is useful in explaining CPI and nominal wage inflation in Hong Kong. A plausible estimate of $u^*_t$ should make the unemployment gap, defined as $u^*_t - u_t$, impact both on $\pi_t$ and $w_t$.

Figure 6 plots the unemployment gap, annual CPI inflation and the output gap $y_t$, which is estimated using a Hodrick-Prescott filter. A clear link between the unemployment gap and $\pi_t$ is evident. Using the data in the figure, we estimate two simple Phillips curves for Hong Kong. We first fit

$$\pi_t = a + b\pi_{t-1} + cy_t + \varepsilon_t. \quad (7)$$

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7See Genberg and Pauwels [2] for a study on price formation in Hong Kong.
To account for potential simultaneity, we estimate all equations with GMM, using the first and second lag of the right-hand side variables as instruments. Table 2 holds the regression output and shows that we do not reject the hypothesis that $b$ equals unity, which suggests that inflation returns only slowly to equilibrium after a shock. Moreover, the constant $a$ is insignificantly different from zero, implying that the long-run mean of inflation is zero.

![Figure 6: Phillips curve data](image)

Note: Sample period 1982:1 to 2003:3

We next include the unemployment gap and estimate

$$\pi_t = a + b\pi_{t-1} + cy_t + d(u^*_t - u_t) + \varepsilon_t.$$  \hspace{1cm} (8)

The impact of the unemployment gap is estimated as 0.58. Thus, when unemployment is one percentage point below its natural level, CPI inflation tends to increase by roughly half a percentage point. The autoregressive coefficient on inflation is significantly smaller than unity, suggesting a comparatively fast return of inflation to equilibrium. More importantly, $a$ is significantly larger than zero. The long-run mean of inflation is estimated as $a/(1 - b) = 6.75\%$. Thus, once the unemployment gap starts to close, we expect to
Table 2: Assessment of the plausibility of $u_t^*$

<table>
<thead>
<tr>
<th>Equation</th>
<th>Phillips curves</th>
<th>Wage equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(7)</td>
<td>(8)</td>
</tr>
<tr>
<td>$a$</td>
<td>-0.142</td>
<td>1.148***</td>
</tr>
<tr>
<td></td>
<td>(0.128)</td>
<td>(0.288)</td>
</tr>
<tr>
<td>$b$</td>
<td>0.982***</td>
<td>0.830***</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>$c$</td>
<td>0.263***</td>
<td>0.061*</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>$d$</td>
<td></td>
<td>0.581***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.190)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.959</td>
<td>0.970</td>
</tr>
</tbody>
</table>

Note: GMM estimates, instruments are two lags of the right-hand side variables. Sample period 1982:4-2003:1 for equations (7) and (8) and 1983:3-2001:3 for equation (9). Standard errors in parentheses. */**/*** denotes significance at the ten / five / one percent level.

observe the emergence of a positive rate of CPI inflation in Hong Kong.

To model wage inflation, we consider the annual increase in nominal wages and the unemployment gap. Figure 7 plots the data and suggests a close relationship between the two variables. This is confirmed by a regression of the form

\[ w_t = a + bw_{t-1} + d(u_t^* - u_t) + \varepsilon_t. \] (9)

The last column of Table 2 shows that the impact of the unemployment gap is significant.\(^8\)

If unemployment is low relative to its natural level, nominal wages tend to increase.

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\(^8\)Including CPI inflation in equation (9) did not yield a significant coefficient.
5 Conclusions

We use the Beveridge curve as analytical framework to estimate the natural rate of unemployment in Hong Kong. We find that the natural rate declined in the 1980s and increased from the beginning of the 1990s onwards. For the third quarter of 2003, when our data end, the natural rate is estimated as three percent and thus lies clearly below the actual unemployment rate of 8.3 percent. This suggests that unemployment is likely to decrease in the near future.

To assess the plausibility of our estimate of the natural rate of unemployment, we fit equations for CPI and wage inflation. We find that the unemployment gap, defined as the difference between the natural and the actual rate of unemployment, is an important explanatory variable.
References


