

Liquidity Risk and the Hedging Role of Options *

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This paper examines the impact of liquidity risk on the behavior of the competitive firm under price uncertainty in a dynamic two-period setting. The firm has access to unbiased one-period futures and option contracts in each period for hedging purposes. A liquidity constraint is imposed on the firm such that the firm is forced to terminate its risk management program in the second period whenever the net loss due to its first-period hedge position exceeds a predetermined threshold level. The imposition of the liquidity constraint on the firm is shown to create perverse incentives to output. Furthermore, the liquidity constrained firm is shown to optimally purchase the unbiased option contracts in the first period if its utility function is quadratic or prudent. This paper thus offers a rationale for the hedging role of options when liquidity risk prevails.

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LIQUIDITY RISK AND THE HEDGING ROLE OF OPTIONS

This paper examines the impact of liquidity risk on the behavior of the competitive firm under price uncertainty in a dynamic two-period setting. The firm has access to unbiased one-period futures and option contracts in each period for hedging purposes. A liquidity constraint is imposed on the firm such that the firm is forced to terminate its risk management program in the second period whenever the net loss due to its first-period hedge position exceeds a predetermined threshold level. The imposition of the liquidity constraint on the firm is shown to create perverse incentives to output. Furthermore, the liquidity constrained firm is shown to optimally purchase the unbiased option contracts in the first period if its utility function is quadratic or prudent. This paper thus offers a rationale for the hedging role of options when liquidity risk prevails.

INTRODUCTION

Liquidity risk can be classified either as asset liquidity risk or as funding liquidity risk (Jorion, 2001). Asset liquidity risk refers to the risk that the liquidation value of the assets differs significantly from the prevailing mark-to-market value. Funding liquidity risk, on the other hand, refers to the risk that payment obligations cannot be met due to inability to raise new funds. Even for firms that are technically solvent, they may be obliged to go bankrupt because of liquidity risk.¹ As such, firms should take liquidity risk seriously when devising their risk management strategies.²

The purpose of this paper is to examine the impact of funding liquidity risk on the behavior of the competitive firm under price uncertainty (Sandmo, 1971) in general, and on the hedging role of options in particular. To this end, Sandmo's (1971) static one-period model is extended to a dynamic two-period setting. Succinctly, the competitive firm has access to unbiased one-period futures and option contracts in each period for hedging purposes. Following Lien (2003) and Wong (2004a, 2004b), a liquidity constraint is imposed on the firm such that the firm is forced to terminate its risk management program

¹Prominent examples include the case of Metallgesellschaft and the debacle of Long-Term Capital Management.

²According to the Committee on Payment and Settlement Systems (1998), liquidity risk is one of the risks that users of derivatives and other financial contracts must take into account.

in the second period whenever the net loss due to its first-period hedge position exceeds a predetermined threshold level. The liquidity constraint as such truncates the firm's payoff profile. This truncation plays a pivotal role in shaping the firm's optimal production and hedging decisions.

In the absence of the liquidity constraint, the well-known separation and full-hedging theorems of Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980) apply. The separation theorem states that the firm's production decision depends neither on the risk attitude of the firm nor on the incidence of the price uncertainty. The full-hedging theorem states that the firm should completely eliminate its price risk exposure by adopting a full-hedge via the unbiased futures contracts. A corollary of the full-hedging theorem is that options play no role as a hedging instrument. The liquidity unconstrained firm uses no options for hedging purposes.

The presence of the liquidity constraint forces the firm to terminate its risk management program in the second period should the net loss due to its first-period hedge position exceed the predetermined threshold level. This creates residual price risk that cannot be hedged via the unbiased futures and option contracts. The firm, being risk averse, is shown to cut down its production as an optimal response to the imposition of the liquidity constraint, a result in line with that of Sandmo (1971). Furthermore, the firm is shown to optimally purchase the unbiased call option contracts in the first period if its utility function is quadratic or is prudent in the sense of Kimball (1990, 1993).³ Since the liquidity constraint truncates the firm's payoff profile, the firm finds the long call option position particularly suitable for its hedging need.

This paper is related to the burgeoning literature on the hedging role of options. Moschini and Lapan (1992) and Wong (2003a) show that export flexibility leads to an ex-ante profit function that is convex in prices. This induced convexity makes options a useful hedging instrument. Sakong, Hayes, and Hallam (1993), Moschini and Lapan (1995),

³The firm's utility function is quadratic or is prudent in the sense of Kimball (1990, 1993) if the marginal utility function is linear or convex, respectively. Unlike risk aversion that is how much one dislikes uncertainty and would turn away from it if one could, prudence measures the propensity to prepare and forearm oneself under uncertainty.

Wong (2003b), and Lien and Wong (2004) show that firms facing both hedgeable and non-hedgeable risks would optimally use options for hedging purposes. The hedging demand for options in this case arises from the fact that the two sources of uncertainty interact in a multiplicative manner, which affects the curvature of profit functions. Lence, Sakong, and Hayes (1994) show that forward-looking firms use options for dynamic hedging purposes because they care about the effects of future output prices on profits from future production cycles. Frechette (2001) demonstrates the value of options in a hedge portfolio when there are transaction costs, even though markets themselves may be unbiased. Futures and options are shown to be highly substitutable and the optimal mix of them are rarely one-sided. Lien and Wong (2002) justify the hedging role of options with multiple delivery specifications in futures markets. The presence of delivery risk creates a truncation of the price distribution, thereby calling for the use of options as a hedging instrument. Chang and Wong (2003) theoretically derive and empirically document the merits of using currency options for cross-hedging purposes, which are due to a triangular parity condition among related spot exchange rates. This paper offers another rationale for the hedging role of options when liquidity risk prevails.

The rest of this paper is organized as follows. The next section delineates a two-period model of the competitive firm facing both price uncertainty and liquidity risk. The firm has access to unbiased one-period futures and option contracts in each period for hedging purposes. The solution to the model is characterized to show the perverse output effect of liquidity constraints. Furthermore, the firm's optimal hedge position in the first period is derived and the hedging role of options is established. A numerical example based on a negative exponential utility function is constructed to help understand the findings. The final section concludes. All proofs of propositions are relegated to the appendix.

THE MODEL

To incorporate liquidity risk into Sandmo's (1971) model of the competitive firm under price

uncertainty, the static one-period set-up is extended into a dynamic one. Succinctly, there are two periods with three dates, indexed by $t = 0, 1$, and 2 . To begin, the firm produces a single output, Q , according to a cost function, $C(Q)$, where $C(0) \geq 0$, $C'(Q) > 0$, and $C''(Q) > 0$. The firm sells its entire output at $t = 2$ at the then prevailing spot price, \tilde{P}_2 , that is not known *ex ante*.⁴ Interest rates in both periods, however, are known with certainty. To simplify notation, the interest factors are henceforth suppressed by compounding all cash flows to their future values at $t = 2$.

To hedge its exposure to the price risk, the firm can trade one-period futures and option contracts at the beginning of each period. The firm is a price taker in the futures and option markets that are populated by many risk-neutral speculators. To rule out arbitrage opportunities, all futures and option contracts are unbiased. Specifically, each of the one-period futures contracts calls for delivery of one unit of the output at the end of the period. By convergence, the futures price at date t must be set equal to the spot price of the output at that time, P_t , where $t = 0, 1$, and 2 . Unbiasedness of the first-period futures contracts implies that P_0 is the expectation of \tilde{P}_1 . Likewise, given the realized spot price of the output at $t = 1$, $\tilde{P}_1 = P_1$, unbiasedness of the second-period futures contracts implies that P_1 is the conditional expectation of \tilde{P}_2 . One can as such specify that $\tilde{P}_2 = P_1 + \tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is a zero-mean random variable conditionally independent of \tilde{P}_1 .

To show the hedging role of options, it suffices to consider only at-the-money call option contracts in each period.⁵ The first-period call option contracts give the holder the right, but not the obligation, to buy one unit of the output per contract at the end of the period at the predetermined exercise price, P_0 . Likewise, given the realized spot price of the output at $t = 1$, $\tilde{P}_1 = P_1$, the second-period call option contracts give the holder the right, but not the obligation, to buy one unit of the output per contract at the end of the period at the predetermined exercise price, P_1 . Unbiasedness of the one-period call option contracts implies that the call option premium, C_t , is set equal to the expectation of $\max(\tilde{P}_{t+1} - P_t, 0)$, where $t = 0$ and 1 .

⁴Throughout the paper, random variables have a tilde (\sim) while their realizations do not.

⁵Put option contracts are not considered because they are redundant in that they can be readily replicated by combinations of futures and call option contracts (Cox & Rubinstein, 1985).

Let H_0 and Z_0 be the numbers of the first-period futures and call option contracts sold (purchased if negative) by the firm at $t = 0$. At $t = 1$, the firm enjoys a net gain (or suffers a net loss if negative) of $(P_0 - P_1)H_0 - \max(P_1 - P_0, 0)Z_0$ from its first-period hedge position, (H_0, Z_0) . Following Lien (2003) and Wong (2004a, 2004b), the firm is liquidity constrained in that it is forced to terminate its risk management program whenever the net loss incurred at $t = 1$ exceeds a predetermined threshold level, K . Thus, if $(P_1 - P_0)H_0 + \max(P_1 - P_0, 0)Z_0 > K$, the firm's random profit at $t = 2$ is given by

$$\tilde{\Pi}_T = \tilde{P}_2 Q + (P_0 - P_1)H_0 + [C_0 - \max(P_1 - P_0, 0)]Z_0 - C(Q). \quad (1)$$

On the other hand, if $(P_1 - P_0)H_0 + \max(P_1 - P_0, 0)Z_0 \leq K$, the firm continues its risk management program in the second period so that its random profit at $t = 2$ becomes

$$\tilde{\Pi}_C = \tilde{\Pi}_T + (P_1 - \tilde{P}_2)H_1 + [C_1 - \max(\tilde{P}_2 - P_1, 0)]Z_1, \quad (2)$$

where $\tilde{\Pi}_T$ is defined in Equation (1), and H_1 and Z_1 are the numbers of the second-period futures and call option contracts sold (purchased if negative) by the firm at $t = 1$.

The firm possesses a von Neumann-Morgenstern utility function, $U(\Pi)$, defined over its profit at $t = 2$, Π , with $U'(\Pi) > 0$ and $U''(\Pi) < 0$, indicating the presence of risk aversion. The firm's multi-period decision problem can be described in the following recursive manner. At $t = 1$, if the net loss from its first-period hedge position, (H_0, Z_0) , does not exceed the threshold level, K , the firm is allowed to choose its second-period hedge position, (H_1, Z_1) , so as to maximize the expected utility of its random profit at $t = 2$, given by Equation (2). At $t = 0$, anticipating the liquidity constraint at $t = 1$ and its second-period optimal hedge position, (H_1^*, Z_1^*) , the firm chooses its output, Q , and its first-period hedge position, (H_0, Z_0) , so as to maximize the expected utility of its random profit at $t = 2$, given by Equations (1) and (2).

SOLUTION TO THE MODEL

As usual, the firm's multi-period decision problem is solved by using backward induction.

At $t = 1$, if $(P_1 - P_0)H_0 + \max(P_1 - P_0, 0)Z_0 > K$, the firm is forced to terminate its risk management program and thereby no further hedging decisions can be made. On the other hand, if $(P_1 - P_0)H_0 + \max(P_1 - P_0, 0)Z_0 \leq K$, the firm is allowed to choose its second-period hedge position, (H_1, Z_1) , so as to maximize the expected utility of its random profit at $t = 2$:

$$\max_{H_1, Z_1} E_\varepsilon[U(\tilde{\Pi}_C)], \quad (3)$$

where $E_\varepsilon(\cdot)$ is the expectation operator with respect to the cumulative distribution function of $\tilde{\varepsilon}$, and $\tilde{\Pi}_C$ is defined in Equation (2). The first-order conditions for program (3) are given by

$$E_\varepsilon[U'(\tilde{\Pi}_C^*)(P_1 - \tilde{P}_2)] = 0, \quad (4)$$

and

$$E_\varepsilon\{U'(\tilde{\Pi}_C^*)[C_1 - \max(\tilde{P}_2 - P_1, 0)]\} = 0, \quad (5)$$

where an asterisk (*) indicates an optimal level.⁶

If $H_1 = Q$ and $Z_1 = 0$, Equation (2) implies that the firm's profit at $t = 2$ becomes $P_1Q + (P_0 - P_1)H_0 + [C_0 - \max(P_1 - P_0, 0)]Z_0 - C(Q)$, which is non-stochastic. Since $P_1 = E_\varepsilon(\tilde{P}_2)$ and $C_1 = E_\varepsilon[\max(\tilde{P}_2 - P_1, 0)]$, it follows that $H_1^* = Q$ and $Z_1^* = 0$ indeed solve Equations (4) and (5) simultaneously. When the firm can continue its risk management program in the second period, there are no more liquidity constraints. In this case, the full-hedging theorem of Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980) applies. As such, the firm finds it optimal to completely eliminate its price risk exposure by adopting a full-hedge via the unbiased second-period futures contracts, i.e., $H_1^* = Q$. There is no hedging role to be played by the second-period call option contracts, i.e., $Z_1^* = 0$.

Now go back to $t = 0$. Let $F(P_1)$ and $f(P_1)$ be the cumulative distribution function and the probability density function of \tilde{P}_1 , respectively, over support $[\underline{P}_1, \overline{P}_1]$, where $0 \leq \underline{P}_1 < \overline{P}_1 \leq \infty$. If $(P_1 - P_0)H_0 + \max(P_1 - P_0, 0)Z_0 \leq K$, i.e., if $P_1 \leq P_0 + K/(H_0 + Z_0)$, the firm anticipates that its optimal second-period hedge position is $(H_1^* = Q, Z_1^* = 0)$ so

⁶The second-order conditions for program (3) are satisfied given risk aversion.

that its random profit at $t = 2$ is given by

$$\Pi_C(P_1) = P_1Q + (P_0 - P_1)H_0 + [C_0 - \max(P_1 - P_0, 0)]Z_0 - C(Q). \quad (6)$$

On the other hand, if $(P_1 - P_0)H_0 + \max(P_1 - P_0, 0)Z_0 < K$, i.e., if $P_1 > P_0 + K/(H_0 + Z_0)$, the firm is forced to terminate its risk management program in the second period so that its random profit at $t = 2$ becomes

$$\Pi_T(P_1, \varepsilon) = (P_1 + \varepsilon)Q + (P_0 - P_1)H_0 + [C_0 - \max(P_1 - P_0, 0)]Z_0 - C(Q). \quad (7)$$

At $t = 0$, the firm's ex-ante decision problem is to choose its output, Q , and its first-period hedge position, (H_0, Z_0) , so as to maximize the expected utility of its random profit at $t = 2$:

$$\max_{Q, H_0, Z_0} \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0 + Z_0}} U[\Pi_C(P_1)] dF(P_1) + \int_{P_0 + \frac{K}{H_0 + Z_0}}^{\bar{P}_1} E_\varepsilon\{U[\Pi_T(P_1, \varepsilon)]\} dF(P_1), \quad (8)$$

where $\Pi_C(P_1)$ and $\Pi_T(P_1, \varepsilon)$ are defined in Equations (6) and (7), respectively. The first-order conditions for program (8) are given by Equations (A.1), (A.2), and (A.3) in the appendix, where Q^* , H_0^* , and Z_0^* are the optimal output, futures position, and call option position, respectively.⁷

OPTIMAL PRODUCTION DECISIONS

This section examines the firm's optimal production decision. As a benchmark, consider the case that the firm faces no liquidity constraints, which is tantamount to setting $K = \infty$. It is evident that the celebrated separation theorem of Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980) applies in this benchmark case. Specifically, the liquidity unconstrained firm's optimal output, Q^0 , solves $C'(Q^0) = P_0$. The following proposition compares Q^* with Q^0 .

⁷The second-order conditions for program (8) are satisfied given risk aversion and the strict convexity of $C(Q)$.

Proposition 1. *If the firm has access to the unbiased one-period futures and call option contracts in each period, imposing the liquidity constraint on the firm creates perverse incentives to output, i.e., $Q^* < Q^0$.*

The intuition of Proposition 1 is as follows. If there are no liquidity constraints, the firm's random profit at $t = 2$ is given by Equation (6) only. In this case, the firm could have completely eliminated its exposure to the price risk had it chosen $H_0 = Q$ and $Z_0 = 0$ within its own discretion. Alternatively put, the degree of risk exposure to be assumed by the firm should be totally unrelated to its production decision. The firm as such chooses its output, Q , to maximize $P_0Q - C(Q)$, which yields Q^0 . This is the celebrated separation theorem of Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980). In the presence of the liquidity constraint, setting $H_0 = Q$ and $Z_0 = 0$ cannot eliminate all the price risk due to the residual risk, $\tilde{\varepsilon}Q$, arising from the termination of the risk management program at $t = 1$, as is evident from Equation (7). Such residual risk, however, can be controlled by varying Q . Given risk aversion, the firm optimally produces less than Q^0 , a result in line with that of Sandmo (1971).

OPTIMAL HEDGING DECISIONS

This section examines the firm's first-period hedging decision. In the benchmark case where the firm faces no liquidity constraints, it is evident that the well-known full-hedging theorem of Danthine (1978), Holthausen (1979), and Feder, Just, and Schmitz (1980) applies. Specifically, the firm's optimal first-period hedge position, (H_0^*, Z_0^*) , satisfies that $H_0^* = Q^*$ and $Z_0^* = 0$, which completely eliminates the price risk exposure to the firm. Thus, the liquidity unconstrained firm uses no options for hedging purposes.

To show the hedging role of options in the presence of liquidity constraints, consider first a rather restrictive case in which $U(\Pi)$ is quadratic, i.e., $U(\Pi) = a\Pi - b\Pi^2$ for some positive scalars, a and b , with $U'(\Pi) = a - 2b\Pi > 0$. The following proposition shows that

the liquidity constrained firm still opts for a full-hedge via the first-period futures contracts, i.e., $H_0^* = Q^*$, but now it also purchases some of the at-the-money call option contracts at $t = 0$ in order to improve the hedging performance.

Proposition 2. *If the liquidity constrained firm has access to the unbiased one-period futures and call option contracts in each period, the firm's optimal first-period hedge position, (H_0^*, Z_0^*) , satisfies that $H_0^* = Q^*$ and $Z_0^* < 0$ when the firm's utility function is quadratic.*

The intuition of Proposition 2 is as follows. Since $\Pi_C^*(P_1) = E_\varepsilon[\Pi_T^*(P_1, \varepsilon)]$ for all $P_1 \in [\underline{P}_1, \overline{P}_1]$ and $U(\Pi)$ is quadratic, it follows from Equations (A.2) and (A.3) that

$$\text{Cov}[\Pi_C^*(\tilde{P}), \tilde{P}_1] = \text{Cov}[\Pi_C^*(\tilde{P}), \max(\tilde{P}_1 - P_0, 0)]. \quad (9)$$

where $\text{Cov}(\cdot, \cdot)$ is the covariance operator with respect to $F(P_1)$. Using the fact that $P_1 - P_0 = \max(P_1 - P_0, 0) - \max(P_0 - P_1, 0)$, we can write Equation (9) as

$$\begin{aligned} \text{Cov}[\Pi_C^*(\tilde{P}), \max(P_0 - \tilde{P}_1, 0)] &= \int_{\underline{P}_1}^{\overline{P}_1} [\Pi_C^*(P_1) - \Pi_C^*(P_0)] \max(P_0 - P_1, 0) dF(P_1) \\ &= \int_{\underline{P}_1}^{P_0} (P_0 - P_1)^2 (H_0^* - Q^*) dF(P_1) = 0, \end{aligned} \quad (10)$$

where the first equality follows from the fact that $\Pi_C^*(P_0)$ equals the expectation of $\Pi_C^*(\tilde{P}_1)$ and the second equality follows from Equation (6) for all $P_1 \in [\underline{P}_1, P_0]$. Equation (10) is satisfied if, and only if, $H_0^* = Q^*$. Substituting $H_0^* = Q^*$ into Equation (A.3) and using the fact that $U(\Pi)$ is quadratic implies that $\text{Var}[\max(\tilde{P}_1 - P_0, 0)]Z_0^* < 0$, where $\text{Var}(\cdot)$ is the variance operator with respect to $F(P_1)$. Thus, it must be the case that $Z_0^* < 0$.

If the firm opts for the first-period hedge position, $(H_0 = Q^*, Z_0 = 0)$, the firm faces no price risk only when its risk management program is continued at $t = 1$, which occurs over the interval, $[\underline{P}_1, P_0 + K/Q^*]$. To further improve the hedging performance, the firm has incentives to purchase the at-the-money call option contracts, i.e., $Z_0 < 0$, so as to enlarge this interval to $[\underline{P}_1, P_0 + K/(Q^* + Z_0)]$. As such, options are used as a hedging instrument when liquidity constraints prevail.

Now relax the unduly restrictive condition of quadratic utility functions. As convincingly argued by Kimball (1990, 1993), prudence, defined as $U'''(\Pi) \geq 0$, is a reasonable behavioral assumption for decision making under multiple sources of uncertainty. Prudence measures the propensity to prepare and forearm oneself under uncertainty, in contrast to risk aversion that is how much one dislikes uncertainty and would turn away from it if one could. As shown by Leland (1968), Drèze and Modigliani (1972), and Kimball (1990), prudence is both necessary and sufficient to induce precautionary saving. Moreover, prudence is implied by decreasing absolute risk aversion, which is instrumental in yielding many intuitively appealing comparative statics under uncertainty (Gollier, 2001). The following proposition shows how the prudent firm would devise its first-period hedge position, (H_0, Z_0) , in the presence of the liquidity constraint.

Proposition 3. *If the liquidity constrained firm has access to the unbiased one-period futures and call option contracts in each period, the firm's optimal first-period hedge position, (H_0^*, Z_0^*) , satisfies that $H_0^* < Q^*$ and $Z_0^* < 0$ when the firm's utility function exhibits prudence.*

The intuition of Proposition 3 is as follows. If the firm opts for the first-period hedge position, $(H_0 = Q^*, Z_0 = 0)$, the firm faces no price risk only when its risk management program is continued at $t = 1$, as is evident from Equations (6) and (7). According to Kimball (1990, 1993), the prudent firm is more sensitive to low realizations of its random profit at $t = 2$ than to high ones. Given that $H_0 = Q^*$ and $Z_0 = 0$, the low realizations of the firm's random profit at $t = 2$ occur when the risk management program is terminated at $t = 1$, which prevails over the interval, $[P_0 + K/Q^*, \bar{P}_1]$, and when the realized values of $\tilde{\varepsilon}$ at $t = 2$ are negative. To avoid these realizations, the firm has incentives to purchase the at-the-money call option contracts, i.e., $Z_0 < 0$, and reduce the first-period futures position, i.e., $H_0 < Q^*$, so as to increase the payoff of its first-period hedge position over the interval, $[P_0, \bar{P}_1]$. Doing so also shrinks the interval, $[P_0 + K/(H_0 + Z_0), \bar{P}_1]$, over which the firm is forced to terminate its risk management program at $t = 1$. Options are particularly useful

for prudent firms facing liquidity constraints because of their asymmetric payoff profiles, vis-à-vis the symmetric payoff profiles of futures. Thus, the prevalence of liquidity constraints offers a rationale for the hedging role of options for prudent firm.

A NUMERICAL EXAMPLE

To gain more insights into the theoretical findings, a numerical example is offered. Suppose that the firm has a negative exponential utility function, $U(\Pi) = -\exp(-\Pi)$, which exhibits prudence. Set $Q = K = 1$, $P_0 = 5$, and $C(Q) \equiv 0$. Let $\tilde{P}_1 = P_0 + \tilde{z}$, where \tilde{z} is independent of $\tilde{\varepsilon}$ and both of which are standard normal variates. Figure 1 depicts the surface of the firm's expected utility as a function of the first-period hedge position, (H_0, Z_0) .

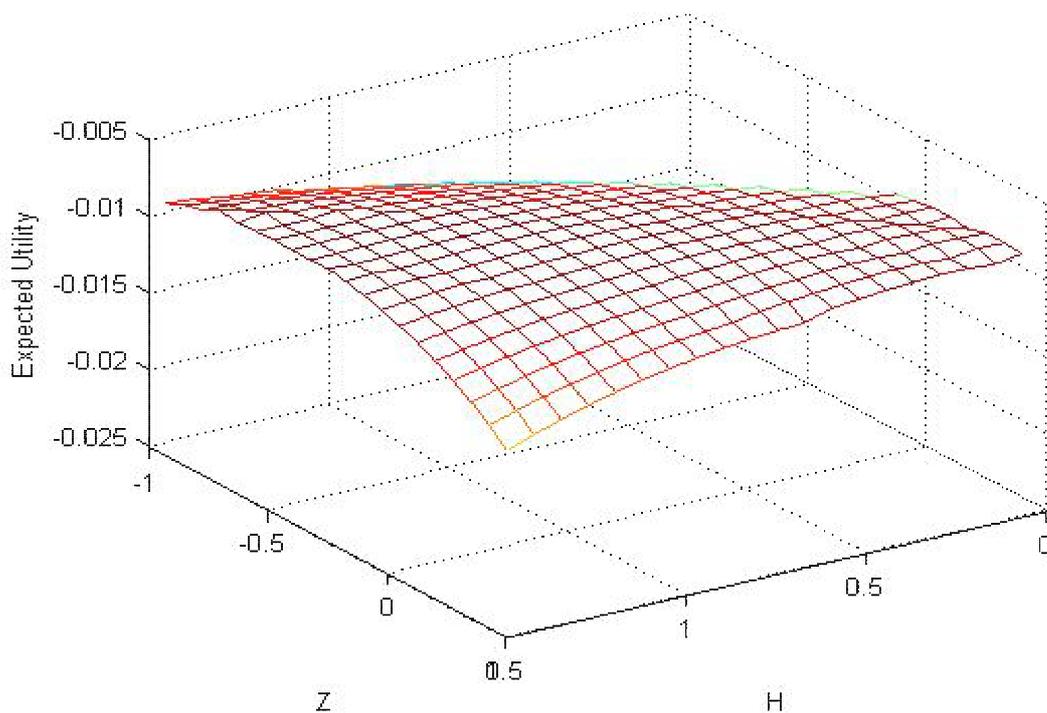


FIGURE 1

Expected utility surface for different futures and call option positions

As is evident from Figure 1, the expected utility surface is rather flat. The maximum is achieved when H_0 and Z_0 are approximately equal to 0.975 and -0.175 , respectively. This is consistent with the results in Proposition 3 that $H_0^* < Q^*$ and $Z_0^* < 0$

CONCLUSIONS

This paper has examined the impact of liquidity risk on the behavior of the competitive firm under price uncertainty *à la* Sandmo (1971) in a dynamic two-period setting. The firm has access to unbiased one-period futures and option contracts in each period for hedging purposes. Following Lien (2003) and Wong (2004a, 2004b), a liquidity constraint is imposed on the firm such that the firm is forced to terminate its risk management program in the second period whenever the net loss due to its first-period hedge position exceeds a predetermined threshold level. The liquidity constraint as such truncates the firm's payoff profile.

The imposition of the liquidity constraint on the firm is shown to create perverse incentives to output. Furthermore, the firm is shown to optimally purchase the unbiased call option contracts in the first period if its utility function is quadratic or is prudent in the sense of Kimball (1990, 1993). Due to their asymmetric payoff profiles, options are particularly suitable for the liquidity constrained firm's hedging need. This paper thus offers a rationale for the hedging role of options when liquidity risk prevails.

APPENDIX

First-Order Conditions for Program (8)

The first-order conditions for program (8) with respect to Q , H_0 , and Z_0 are respectively given by

$$\int_{\underline{P}_1}^{P_0 + \frac{K}{H_0^* + Z_0^*}} U'[\Pi_C^*(P_1)][P_1 - C'(Q^*)] dF(P_1) + \int_{P_0 + \frac{K}{H_0^* + Z_0^*}}^{\bar{P}_1} E_\varepsilon\{U'[\Pi_T^*(P_1, \tilde{\varepsilon})][P_1 + \tilde{\varepsilon} - C'(Q^*)]\} dF(P_1) = 0, \quad (\text{A.1})$$

$$\begin{aligned} & \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0^* + Z_0^*}} U'[\Pi_C^*(P_1)](P_0 - P_1) dF(P_1) \\ & + \int_{P_0 + \frac{K}{H_0^* + Z_0^*}}^{\bar{P}_1} E_\varepsilon\{U'[\Pi_T^*(P_1, \tilde{\varepsilon})]\}(P_0 - P_1) dF(P_1) \\ & + \left\{ E_\varepsilon \left\{ U \left[\Pi_T^* \left(P_0 + \frac{K}{H_0^* + Z_0^*}, \tilde{\varepsilon} \right) \right] \right\} - U \left[\Pi_C^* \left(P_0 + \frac{K}{H_0^* + Z_0^*} \right) \right] \right\} \\ & \times f \left(P_0 + \frac{K}{H_0^* + Z_0^*} \right) \frac{K}{(H_0^* + Z_0^*)^2} = 0, \end{aligned} \quad (\text{A.2})$$

and

$$\begin{aligned} & \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0^* + Z_0^*}} U'[\Pi_C^*(P_1)][C_0 - \max(P_1 - P_0, 0)] dF(P_1) \\ & + \int_{P_0 + \frac{K}{H_0^* + Z_0^*}}^{\bar{P}_1} E_\varepsilon\{U'[\Pi_T^*(P_1, \tilde{\varepsilon})]\}[C_0 - \max(P_1 - P_0, 0)] dF(P_1) \\ & + \left\{ E_\varepsilon \left\{ U \left[\Pi_T^* \left(P_0 + \frac{K}{H_0^* + Z_0^*}, \tilde{\varepsilon} \right) \right] \right\} - U \left[\Pi_C^* \left(P_0 + \frac{K}{H_0^* + Z_0^*} \right) \right] \right\} \\ & \times f \left(P_0 + \frac{K}{H_0^* + Z_0^*} \right) \frac{K}{(H_0^* + Z_0^*)^2} = 0, \end{aligned} \quad (\text{A.3})$$

where Equations (A.2) and (A.3) follow from Leibniz's rule, and an asterisk (*) signifies an optimal level.

Proof of Proposition 1

Adding Equation (A.2) to Equation (A.1) and rearranging terms yields

$$\begin{aligned}
\mathbb{E}[U'(\tilde{\Pi}^*)][P_0 - C'(Q^*)] &= - \int_{P_0 + \frac{K}{H_0^* + Z_0^*}}^{\bar{P}_1} \mathbb{E}_\varepsilon\{U'[\Pi_T^*(P_1, \tilde{\varepsilon})]\tilde{\varepsilon}\} dF(P_1) \\
&+ \left\{ U \left[\Pi_C^* \left(P_0 + \frac{K}{H_0^* + Z_0^*} \right) \right] - \mathbb{E}_\varepsilon \left\{ U \left[\Pi_T^* \left(P_0 + \frac{K}{H_0^* + Z_0^*}, \tilde{\varepsilon} \right) \right] \right\} \right\} \\
&\times f \left(P_0 + \frac{K}{H_0^* + Z_0^*} \right) \frac{K}{(H_0^* + Z_0^*)^2}. \tag{A.4}
\end{aligned}$$

where $\mathbb{E}[U'(\tilde{\Pi}^*)] = \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0^* + Z_0^*}} U'[\Pi_C^*(P_1)] dF(P_1) + \int_{P_0 + \frac{K}{H_0^* + Z_0^*}}^{\bar{P}_1} \mathbb{E}_\varepsilon\{U'[\Pi_T^*(P_1, \tilde{\varepsilon})]\} dF(P_1)$. If the firm faces no liquidity constraints, which is tantamount to setting $K = \infty$, it is evident that the right-hand side of Equation (A.4) vanishes. Thus, in this hypothetical case, the firm's optimal output, Q^0 , solves $C'(Q^0) = P_0$.

Now resume the original case wherein the firm faces the liquidity constraint, i.e., $K < \infty$. Note that $\mathbb{E}_\varepsilon\{U'[\Pi_T^*(P_1, \tilde{\varepsilon})]\tilde{\varepsilon}\} = \text{Cov}_\varepsilon\{U'[\Pi_T^*(P_1, \tilde{\varepsilon})], \tilde{\varepsilon}\}$, where $\text{Cov}_\varepsilon(\cdot, \cdot)$ is the covariance operator with respect to the cumulative distribution function of $\tilde{\varepsilon}$. Note also that $\partial U'[\Pi_T^*(P_1, \varepsilon)]/\partial \varepsilon = U''[\Pi_T^*(P_1, \varepsilon)]Q^* < 0$. Thus, the first term on the right-hand side of Equation (A.4) is positive. Using Equations (6) and (7) and the fact that $\tilde{\varepsilon}$ has a mean of zero yields $\Pi_C^*[P_0 + K/(H_0^* + Z_0^*)] = \mathbb{E}_\varepsilon\{\Pi_T^*[P_0 + K/(H_0^* + Z_0^*), \tilde{\varepsilon}]\}$. It then follows from Jensen's inequality and risk aversion that the expression inside the curly brackets of the second term on the right-hand of Equation (A.4) is positive. Thus, the right-hand of Equation (A.4) is unambiguously positive so that $C'(Q^*) < P_0$. The strict convexity of $C(Q)$ then implies that $Q^* < Q^0$.

Proof of Proposition 2

Subtracting Equation (A.2) by Equation (A.3) yields

$$\begin{aligned} & \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0^* + Z_0^*}} U'[\Pi_C^*(P_1)] [\max(P_0 - P_1, 0) - C_0] dF(P_1) \\ & + \int_{P_0 + \frac{K}{H_0^* + Z_0^*}}^{\bar{P}_1} \mathbb{E}_\varepsilon \{U'[\Pi_T^*(P_1, \tilde{\varepsilon})]\} [\max(P_0 - P_1, 0) - C_0] dF(P_1) = 0, \end{aligned} \quad (\text{A.5})$$

which follows from the fact that $P_0 - P_1 = \max(P_0 - P_1, 0) - \max(P_1 - P_0, 0)$. Since P_0 and C_0 are the expectations of \tilde{P}_1 and $\max(\tilde{P}_1 - P_0, 0)$, respectively, C_0 is also equal to the expectation of $\max(P_0 - \tilde{P}_1, 0)$.

Define the following:

$$M = \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0^* + Z_0^*}} U'[\Pi_C^*(P_1)] dF(P_1) + \int_{P_0 + \frac{K}{H_0^* + Z_0^*}}^{\bar{P}_1} \mathbb{E}_\varepsilon \{U'[\Pi_T^*(P_1, \tilde{\varepsilon})]\} dF(P_1), \quad (\text{A.6})$$

where M is simply the firm's expected marginal utility at the optimum. Since C_0 equals the expectation of $\max(P_0 - \tilde{P}_1, 0)$, one can use Equation (A.6) to write Equation (A.5) as

$$\begin{aligned} & \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0^* + Z_0^*}} \{U'[\Pi_C^*(P_1)] - M\} [\max(P_0 - P_1, 0) - C_0] dF(P_1) \\ & + \int_{P_0 + \frac{K}{H_0^* + Z_0^*}}^{\bar{P}_1} \left\{ \mathbb{E}_\varepsilon \{U'[\Pi_T^*(P_1, \tilde{\varepsilon})]\} - M \right\} [\max(P_0 - P_1, 0) - C_0] dF(P_1) = 0. \end{aligned} \quad (\text{A.7})$$

Inspection of Equations (A.6) and (A.7) reveals that Equation (A.7) can be simplified to

$$\int_{\underline{P}_1}^{P_0} \{U'[\Pi_C^*(P_1)] - M\} (P_0 - P_1) dF(P_1) = 0. \quad (\text{A.8})$$

Since $U(\Pi)$ is quadratic and $\tilde{\varepsilon}$ has a mean of zero, Equation (A.6) implies that $M = U'[\Pi_C^*(P_0)]$. It follows from Equation (6) that $\Pi_C^*(P_1) = P_1(Q^* - H_0^*) + P_0 H_0^* + C_0 Z_0^* - C(Q^*)$ for all $P_1 \in [\underline{P}_1, P_0]$. Suppose that $H_0^* > (<) Q^*$. Then, $M > (<) U'[\Pi_C^*(P_1)]$ for all $P_1 \in [\underline{P}_1, P_0]$ so that the left-hand side of Equation (A.7) is negative (positive), a contradiction. Hence, it must be the case that $H_0^* = Q^*$ when the firm has a quadratic utility function.

Using Equation (A.6), Equation (A.3) can be written as

$$\begin{aligned}
& \int_{P_0}^{P_0 + \frac{K}{H_0^* + Z_0^*}} \{M - U'[\Pi_C^*(P_1)]\} (P_1 - P_0) dF(P_1) \\
& + \int_{P_0 + \frac{K}{H_0^* + Z_0^*}}^{\bar{P}_1} \left\{ M - E_\varepsilon \{U'[\Pi_T^*(P_1, \tilde{\varepsilon})]\} \right\} (P_1 - P_0) dF(P_1) \\
& + \left\{ E_\varepsilon \left\{ U \left[\Pi_T^* \left(P_0 + \frac{K}{H_0^* + Z_0^*}, \tilde{\varepsilon} \right) \right] \right\} - U \left[\Pi_C^* \left(P_0 + \frac{K}{H_0^* + Z_0^*} \right) \right] \right\} \\
& \times f \left(P_0 + \frac{K}{H_0^* + Z_0^*} \right) \frac{K}{(H_0^* + Z_0^*)^2} = 0, \tag{A.9}
\end{aligned}$$

The final term on the left-hand side of Equation (A.9) is negative given risk aversion. If the firm's utility function is quadratic, it follows that $U'[\Pi_C^*(P_1)] = E_\varepsilon \{U'[\Pi_T^*(P_1, \tilde{\varepsilon})]\}$ and $M = U'[\Pi_C^*(P_0)]$. Hence, the first two term on the left-hand side of Equation (A.9) can be written as

$$\int_{P_0}^{\bar{P}_1} \{U'[\Pi_C^*(P_0)] - U'[\Pi_C^*(P_1)]\} (P_1 - P_0) dF(P_1). \tag{A.10}$$

From Equation (6) and the fact that $H_0^* = Q^*$, it must be the case that $\Pi_C^*(P_1) = \Pi_C^*(P_0) - (P_1 - P_0)Z_0^*$ for all $P_1 \in (P_0, \bar{P}_1]$. Suppose that $Z_0^* \geq 0$. Then, $\Pi_C^*(P_1) \leq \Pi_C^*(P_0)$ for all $P_1 \in (P_0, \bar{P}_1]$ so that expression (A.10) is non-positive. This implies that the left-hand side of Equation (A.9) is negative, a contradiction. Hence, it must be the case that $Z_0^* < 0$ when the firm's utility function is quadratic.

Proof of Proposition 3

Using Equations (6) and (7) and the fact that $\tilde{\varepsilon}$ has a mean of zero yields $\Pi_C^*(P_1) = E_\varepsilon[\Pi_T^*(P_1, \tilde{\varepsilon})]$. It then follows from Jensen's inequality and prudence that $U'[\Pi_C^*(P_1)] < E_\varepsilon\{U'[\Pi_T^*(P_1, \tilde{\varepsilon})]\}$. Hence, Equation (A.6) implies that

$$M > \int_{\underline{P}_1}^{\bar{P}_1} U'[\Pi_C^*(P_1)] dF(P_1) > U'[\Pi_C^*(P_0)], \tag{A.11}$$

where the second inequality follows from prudence and the fact that $\Pi_C^*(P_0)$ equals the expectation of $\Pi_C^*(\tilde{P}_1)$. Suppose that $H_0^* \geq Q^*$. Then, $\partial U'[\Pi_C^*(P_1)]/\partial P_1 = U''[\Pi_C^*(P_1)](Q^* -$

$H_0^* \geq 0$ for all $P_1 \in [\underline{P}_1, P_0]$ so that $M > U'[\Pi_C^*(P_1)]$. In this case, the right-hand side of Equation (A.8) is negative, a contradiction. Hence, it must be the case that $H_0^* < Q^*$ when the firm is prudent.

To facilitate the proof that $Z_0^* < 0$, the firm's ex-ante decision problem at $t = 0$ is reformulated as a two-stage optimization problem, where the firm's output is fixed at Q^* . In the first stage, the firm's demand for the first-period futures contracts, $H_0(Z_0)$, is derived for a given call option position, Z_0 . In the second stage, the firm's optimal call option position, Z_0^* , is solved taking $H_0(Z_0)$ as given. The complete solution to the firm's ex-ante decision problem is thus $H_0^* = H_0(Z_0^*)$ and Z_0^* .

Using Leibniz's rule, the first-order condition for the first-stage optimization problem is given by

$$\begin{aligned} & \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0(Z_0) + Z_0}} U'[\hat{\Pi}_C(P_1, Z_0)](P_0 - P_1) dF(P_1) \\ & + \int_{P_0 + \frac{K}{H_0(Z_0) + Z_0}}^{\bar{P}_1} E_\varepsilon\{U'[\hat{\Pi}_T(P_1, \tilde{\varepsilon}, Z_0)]\}(P_0 - P_1) dF(P_1) \\ & + \left\{ E_\varepsilon\left\{ U\left[\hat{\Pi}_T\left(P_0 + \frac{K}{H_0(Z_0) + Z_0}, \tilde{\varepsilon}, Z_0 \right) \right] \right\} - U\left[\hat{\Pi}_C\left(P_0 + \frac{K}{H_0(Z_0) + Z_0}, Z_0 \right) \right] \right\} \\ & \times f\left[P_0 + \frac{K}{H_0(Z_0) + Z_0} \right] \frac{K}{[H_0(Z_0) + Z_0]^2} = 0, \end{aligned} \quad (\text{A.12})$$

where $\hat{\Pi}_C(P_1, Z_0)$ and $\hat{\Pi}_T(P_1, \varepsilon, Z_0)$ are defined in Equations (6) and (7) with $Q = Q^*$ and $H_0 = H_0(Z_0)$, respectively. The second-order condition for the first-stage optimization problem is satisfied given risk aversion.

When $Z_0 = 0$, if $H_0(0) = Q^*$, the left-hand side of Equation (A.12) becomes

$$\begin{aligned} & \left\{ U'[\hat{\Pi}_C(P_0, 0)] - E_\varepsilon\{U'[\hat{\Pi}_T(P_0, \tilde{\varepsilon}, 0)]\} \right\} \int_{P_0 + \frac{K}{Q^*}}^{\bar{P}_1} (P_1 - P_0) dF(P_1) \\ & + \left\{ E_\varepsilon\{U[\hat{\Pi}_T(P_0, \tilde{\varepsilon}, 0)]\} - U[\hat{\Pi}_C(P_0, 0)] \right\} f\left(P_0 + \frac{K}{Q^*} \right) \frac{K}{Q^{*2}}, \end{aligned} \quad (\text{A.13})$$

where the first term of expression (A.13) follows from the fact that P_0 is the expecta-

tion of \tilde{P}_1 . Using Equations (6) and (7) and the fact that $\tilde{\varepsilon}$ has a mean of zero yields $\hat{\Pi}_C(P_0, 0) = E_\varepsilon[\hat{\Pi}_T(P_0, \tilde{\varepsilon}, 0)]$. It follows from Jensen's inequality that $U[\hat{\Pi}_C(P_0, 0)] > E_\varepsilon\{U[\hat{\Pi}_T(P_0, \tilde{\varepsilon}, 0)]\}$ and $U'[\hat{\Pi}_C(P_0, 0)] < E_\varepsilon\{U'[\hat{\Pi}_T(P_0, \tilde{\varepsilon}, 0)]\}$. Thus, expression (A.13) is negative, implying that $H_0(0) < Q^*$.

Now proceed to the second-stage optimization problem of the firm, which can be stated as

$$\begin{aligned} \max_{Z_0} V(Z_0) &= \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0(Z_0) + Z_0}} U[\hat{\Pi}_C(P_1, Z_0)] dF(P_1) \\ &\quad + \int_{P_0 + \frac{K}{H_0(Z_0) + Z_0}}^{\bar{P}_1} E_\varepsilon\{U[\hat{\Pi}_T(P_1, \tilde{\varepsilon}, Z_0)]\} dF(P_1). \end{aligned} \quad (\text{A.14})$$

Totally differentiating Equation (A.14) with respect to Z_0 and evaluating the resulting derivative at $Z_0 = 0$ yields

$$\begin{aligned} V'(0) &= \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0(0)}} U'[\hat{\Pi}_C(P_1, 0)][C_0 - \max(P_1 - P_0, 0)] dF(P_1) \\ &\quad + \int_{P_0 + \frac{K}{H_0(0)}}^{\bar{P}_1} E_\varepsilon\{U'[\hat{\Pi}_T(P_1, \tilde{\varepsilon}, 0)]\}[C_0 - \max(P_1 - P_0, 0)] dF(P_1) \\ &\quad + \left\{ E_\varepsilon\left\{ U\left[\hat{\Pi}_T\left(P_0 + \frac{K}{H_0(0)}, \tilde{\varepsilon}, 0 \right) \right] \right\} - U\left[\hat{\Pi}_C\left(P_0 + \frac{K}{H_0(0)}, 0 \right) \right] \right\} \\ &\quad \times f\left[P_0 + \frac{K}{H_0(0)} \right] \frac{K}{H_0(0)^2}, \end{aligned} \quad (\text{A.15})$$

which follows from the envelope theorem and Leibniz's rule. Substituting Equation (A.12) with $Z_0 = 0$ into the right-hand side of Equation (A.15) yields

$$\begin{aligned} V'(0) &= \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0(0)}} U'[\hat{\Pi}_C(P_1, 0)][C_0 - \max(P_0 - P_1, 0)] dF(P_1) \\ &\quad + \int_{P_0 + \frac{K}{H_0(0)}}^{\bar{P}_1} E_\varepsilon\{U'[\hat{\Pi}_T(P_1, \tilde{\varepsilon}, 0)]\}[C_0 - \max(P_0 - P_1, 0)] dF(P_1), \end{aligned} \quad (\text{A.16})$$

which follows from the fact that $P_0 - P_1 = \max(P_0 - P_1, 0) - \max(P_1 - P_0, 0)$.

Define the following:

$$N = \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0(0)}} U'[\hat{\Pi}_C(P_1, 0)] dF(P_1) + \int_{P_0 + \frac{K}{H_0(0)}}^{\bar{P}_1} E_\varepsilon\{U'[\hat{\Pi}_T(P_1, \tilde{\varepsilon}, 0)]\} dF(P_1). \quad (\text{A.17})$$

Using Equation (A.17), Equation (A.16) can be written as

$$\begin{aligned} V'(0) &= \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0(0)}} \{U'[\hat{\Pi}_C(P_1, 0)] - N\} [C_0 - \max(P_0 - P_1, 0)] dF(P_1) \\ &+ \int_{P_0 + \frac{K}{H_0(0)}}^{\bar{P}_1} \left\{ E_\varepsilon\{U'[\hat{\Pi}_T(P_1, \tilde{\varepsilon}, 0)]\} - N \right\} [C_0 - \max(P_0 - P_1, 0)] dF(P_1), \end{aligned} \quad (\text{A.18})$$

since C_0 equals the expectation of $\max(P_0 - \tilde{P}_1, 0)$. Inspection of Equations (A.17) and (A.18) reveals that Equation (A.18) can be simplified to

$$V'(0) = \int_{\underline{P}_1}^{P_0} \{N - U'[\hat{\Pi}_C(P_1, 0)]\} (P_0 - P_1) dF(P_1). \quad (\text{A.19})$$

Using Equations (6) and (7) and the fact that $\tilde{\varepsilon}$ has a mean of zero yields $\hat{\Pi}_C(P_1, 0) = E_\varepsilon[\hat{\Pi}_T(P_1, \tilde{\varepsilon}, 0)]$. It follows from prudence and Jensen's inequality that $U'[\hat{\Pi}_C(P_1, 0)] < E_\varepsilon\{U'[\hat{\Pi}_T(P_0, \tilde{\varepsilon}, 0)]\}$. Thus, Equation (A.17) implies that

$$N > \int_{\underline{P}_1}^{\bar{P}_1} U'[\hat{\Pi}_C(P_1, 0)] dF(P_1) > U'[\hat{\Pi}_C(P_0, 0)], \quad (\text{A.20})$$

where the second inequality follows from prudence and the fact that $\hat{\Pi}_C(P_0, 0)$ equals the expectation of $\hat{\Pi}_C(\tilde{P}_1, 0)$. Since $H_0(0) < Q^*$, $\partial U'[\hat{\Pi}_C(P_1, 0)]/\partial P_1 = U''[\hat{\Pi}_C(P_1, 0)][Q^* - H_0(0)] < 0$ for all $P_1 \in [\underline{P}_1, P_0]$. Thus, it follows from inequality (A.20) that there must exist a unique point, $\hat{P}_1 \in (\underline{P}_1, P_0)$, such that $N = U'[\hat{\Pi}_C(\hat{P}_1, 0)]$. Using Equation (A.17), Equation (A.19) can be written as

$$\begin{aligned} V'(0) &= \int_{\underline{P}_1}^{P_0} \{N - U'[\hat{\Pi}_C(P_1, 0)]\} (\hat{P}_1 - P_1) dF(P_1) \\ &+ \int_{\underline{P}_1}^{P_0} \{N - U'[\hat{\Pi}_C(P_1, 0)]\} dF(P_1) (P_0 - \hat{P}_1). \end{aligned} \quad (\text{A.21})$$

Since $U'[\hat{\Pi}_C(P_1, 0)] > (<) N$ whenever $P_1 < (>) \hat{P}_1$, the first term on the right-hand side of Equation (A.21) is positive. Define the following:

$$N_H = \int_{\underline{P}_1}^{P_0 + \frac{K}{H_0(0)}} U'[\hat{\Pi}_C(P_1, 0)] dF(P_1) / F\left[P_0 + \frac{K}{H_0(0)}\right]. \quad (\text{A.22})$$

Note that, for all $P_1 \in [\underline{P}_1, \bar{P}_1]$, $\partial U'[\hat{\Pi}_C(P_1, 0)]/\partial P_1 = U''[\hat{\Pi}_C(P_1, 0)][Q^* - H_0(0)] < 0$ and $\partial E_\varepsilon\{U'[\hat{\Pi}_T(P_1, \tilde{\varepsilon}, 0)]\}/\partial P_1 = E_\varepsilon\{U''[\hat{\Pi}_T(P_1, \tilde{\varepsilon}, 0)]\}[Q^* - H_0(0)] < 0$ since $H_0(0) < Q^*$. It follows that $N_H > N$ as N_H is computed over $[\underline{P}_1, P_0 + K/H_0(0)]$, the low realizations of \tilde{P}_1 . Using Equation (A.22) yields

$$\begin{aligned} & \int_{\underline{P}_1}^{P_0} \{N - U'[\hat{\Pi}_C(P_1, 0)]\} dF(P_1) \\ &= F\left[P_0 + \frac{K}{H_0(0)}\right](N - N_H) + \int_{P_0}^{P_0 + \frac{K}{H_0(0)}} \{U'[\hat{\Pi}_C(P_1, 0)] - N\} dF(P_1). \quad (\text{A.23}) \end{aligned}$$

The first term on the right-hand side of Equation (A.23) is negative because $N_H > N$. The second term is also negative because $N > U'[\hat{\Pi}_C(P_1, 0)]$ for all $P_1 \in [P_0, P_0 + K/H_0(0)]$. Hence, the second term on the right-hand side of Equation (A.21) is negative. Since $V'(0) < 0$, it must be the case that $Z_0^* < 0$ when the firm's utility function exhibits prudence.

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