Foreign Direct Investment and Currency Hedging

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This paper examines the behavior of a risk-averse multinational firm (MNF) under exchange rate uncertainty. The MNF has an investment opportunity in a foreign country. To hedge the exchange rate risk, the MNF can avail itself of customized derivative contracts that are fairly priced. Foreign direct investment (FDI) is irreversible and costly expandable in that the MNF can acquire additional capital at a higher unit price after the spot exchange rate has been publicly revealed. The MNF as such possesses a real (call) option that is rationally exercised whenever the foreign currency has been substantially appreciated relative to the domestic currency. The ex-post exercise of the real option convexifies the MNF's ex-ante domestic currency profit with respect to the random spot exchange rate, thereby calling for the use of currency options as a hedging instrument. We show that the MNF's optimal initial level of sequential FDI is always lower than that of lumpy FDI, while the expected optimal aggregate level of sequential FDI can be higher or lower than that of lumpy FDI. We further show that currency hedging, no matter perfect or imperfect, improves the MNF's ex-ante and ex-post incentives to make FDI, a result consistent with the complementary nature of operational and financial hedging strategies.

JEL classification: D81; F23; F31

Keywords: Foreign direct investment; Real options; Currency hedging

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Foreign direct investment and currency hedging

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Abstract

This paper examines the behavior of a risk-averse multinational firm (MNF) under exchange rate uncertainty. The MNF has an investment opportunity in a foreign country. To hedge the exchange rate risk, the MNF can avail itself of customized derivative contracts that are fairly priced. Foreign direct investment (FDI) is irreversible and costly expandable in that the MNF can acquire additional capital at a higher unit price after the spot exchange rate has been publicly revealed. The MNF as such possesses a real (call) option that is rationally exercised whenever the foreign currency has been substantially appreciated relative to the domestic currency. The ex-post exercise of the real option convexifies the MNF's ex-ante domestic currency profit with respect to the random spot exchange rate, thereby calling for the use of currency options as a hedging instrument. We show that the MNF's optimal initial level of sequential FDI is always lower than that of lumpy FDI, while the expected optimal aggregate level of sequential FDI can be higher or lower than that of lumpy FDI. We further show that currency hedging, no matter perfect or imperfect, improves the MNF's ex-ante and ex-post incentives to make FDI, a result consistent with the complementary nature of operational and financial hedging strategies.

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1. Introduction

Foreign direct investment (FDI) is a sequential process that determines the volume and direction of resources transferred across borders (Kogut, 1983). The ability of multinational firms (MNFs) to arbitrage institutional restrictions (e.g., tax codes, relative exchange rates,

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and remittance forms) creates a string of options that are written on various contingent outcomes. In this regard, MNFs are best described as a collection of valuable options that permit discretionary choices among alternative real economic activities and financial flows from one country to the other.

One important strand of the literature on MNFs under exchange rate uncertainty focuses on the effect of currency hedging on the behavior of MNFs. (see, e.g., Broll, 1992; Broll and Zilcha, 1992; Broll, Wong, and Zilcha, 1999; Chang and Wong, 2003; Wong, 2003a). The typical scenario is that a risk-averse MNF makes its FDI and hedging decisions simultaneously prior to the resolution of the exchange rate uncertainty. Two notable results emanate. First, the separation theorem states that the MNF's optimal FDI decision is affected neither by its risk attitude nor by the underlying exchange rate uncertainty when there is a currency forward/futures market. Second, the full-hedging theorem states that the MNF optimally opts for a full-hedge to completely eliminate its exchange rate risk exposure should the currency forward/futures market be unbiased.

Taking FDI as a sequential process into account, we depart from the extant literature by allowing the MNF to make sequential, rather than lumpy, FDI decisions. To this end, the MNF has the right, but not the obligation, to alter its level of FDI after the exchange rate uncertainty has been completely resolved. We model FDI to be irreversible and costly expandable in that the MNF can purchase additional, but cannot sell redundant, capital in the domestic country at a higher unit price of capital when adjustments in FDI are called for.[1] The flexibility of making sequential FDI, vis-à-vis lumpy FDI, proffers the MNF a real (call) option that is rationally exercised whenever the foreign currency has been substantially appreciated relative to the domestic currency. The ex-post exercise of the real option as such convexifies the MNF's ex-ante domestic currency profit with respect to the random spot exchange rate.

To examine how the MNF's optimal FDI decisions are affected by the interaction between operational and financial hedging, we allow the MNF to avail itself of fairly priced

[1] Dixit and Pindyck (1994) argue that asset specificity, information asymmetry, and government regulations are plausible reasons why FDI is irreversible and costly expandable. See also Wong (2006).

derivatives that can be tailor-made for its hedging need. We show that the MNF optimally tailors its customized derivative contract in a way that its hedged domestic currency profit is stabilized at the expected level, thereby eliminating all the exchange rate risk. The MNF's optimal initial level of FDI is thus identical to the one when the MNF is risk neutral, which is always lower than the optimal level of lumpy FDI. The expected optimal aggregate level of sequential FDI, however, can be higher or lower than that of lumpy FDI. We further show that the MNF's optimal customized derivative contract can be perfectly replicated by trading the unbiased currency futures and a continuum of fairly priced currency put and call option contracts of all exercise prices, a result consistent with the prevalent use of currency options by non-financial firms (Bodnar, Hayt, and Marston, 1998).

If the MNF is banned from engaging in currency hedging, we show that the MNF's ex-ante and ex-post incentives to make FDI are reduced as compared to those with perfect currency hedging. Since a financial hedge is absent, the MNF has to rely on an operational hedge via lowing its FDI. We further show that an increase in the fixed or setup cost incurred by the MNF gives rise to similar perverse effects on FDI should the MNF's risk preferences exhibit the reasonable property of decreasing absolute risk aversion. Given that the change in the fixed or setup cost may be due to a change in the investment tax credits offered by the host government, or due to a change in the severity of entry barriers in the host country, FDI flows are expected to react in a predictable manner when these government policies and market conditions shift over time. These implications are largely consistent with the empirical findings of Anand and Kogut (1997) and Hines (2001).

If the MNF is restricted to use the unbiased currency futures contracts as the sole hedging instrument, we show that risk aversion has no effect on the expected marginal return to the initial level of FDI, but has a negative effect on the option value of waiting to make subsequent FDI. The former is due to the spanning property that arises from the tradability of the random spot exchange rate via trading the unbiased currency futures contracts. The latter is due to the non-tradability of the real option embedded in sequential FDI so that spanning is not possible, making the MNF's risk preferences impact negatively on the pricing of the option in this incomplete market context. The MNF's ex-ante and ex-post incentives to make FDI are therefore enhanced as compared to those with perfect currency hedging. This implies immediately that futures hedging promotes FDI, a result consistent with the complementary nature of operational and financial hedging strategies (Allayannis, Ihrig, and Weston, 2001; Kim, Mathur, and Nam, 2006).

The rest of this paper is organized as follows. Section 2 delineates a dynamic model of a risk-averse MNF that makes sequential FDI decisions in response to the intertemporal resolution of exchange rate uncertainty. Section 3 derives the MNF's optimal FDI and hedging decisions when there are fairly priced derivatives that can be tailor-made for the MNF's hedging need. Section 4 compares the MNF's optimal FDI decisions in the case of sequential FDI to those in the case of lumpy FDI. Section 5 examines the effects of banning the MNF from engaging in currency hedging on its optimal sequential FDI decisions. Section 6 restricts the MNF to use the unbiased currency futures contracts as the sole hedging instrument and shows that futures hedging improves the MNF's ex-ante and ex-post incentives to make FDI. The final section concludes.

2. The model

Consider a multinational firm (MNF) that invests in a foreign country under exchange rate uncertainty. There is one period with three dates, indexed by $t = 0$, 1, and 2. The prevailing spot exchange rate at $t = 2$, which is denoted by $\tilde{e}$ and is expressed in units of the domestic currency per unit of the foreign currency, is uncertain at $t = 0$.[2] The MNF regards $\tilde{e}$ as a positive random variable distributed according to a known cumulative distribution function, $F(e)$, over support $[\underline{e}, \overline{e}]$, where $0 \leq \underline{e} < \overline{e} \leq \infty$.[3] The exchange rate uncertainty, however, is completely resolved at $t = 1$, at which time the true realization of $\tilde{e}$ is publicly

[2]Throughout the paper, random variables have a tilde ($\tilde{}$) while their realizations do not.

[3]An alternative way to model the exchange rate uncertainty is to apply the concept of information systems that are conditional cumulative distribution functions over a set of signals imperfectly correlated with $\tilde{e}$ (Eckwert and Zilcha, 2001, 2003; Drees and Eckwert, 2003; Broll and Eckwert, 2006). The advantage of this more general and realistic approach is that one can study the value of information by comparing the information content of different information systems. Since the focus of this paper is not on the value of information, we adopt a simpler structure to save notation.

observed. The riskless rate of interest is known and constant for the period. To simplify notation, we henceforth suppress the interest factors by compounding all cash flows to their future values at $t = 2$.

To begin, the MNF incurs a fixed cost, $c \geq 0$, for the access to a project in the foreign country. If the MNF makes foreign direct investment (FDI) of k units of capital that are acquired in the home country, the project yields a deterministic cash flow of $f(k)$ at $t = 2$, where $f(k)$ is denominated in the foreign currency with $f(0) = 0$, $f'(k) > 0$ and $f''(k) < 0$ for all $k \geq 0$, $\lim_{k\to 0} f'(k) = \infty$, and $\lim_{k\to\infty} f'(k) = 0$. FDI has the properties of being sequential, irreversible, and costly expandable. Succinctly, at $t = 0$, the MNF acquires k_0 units of capital at a known unit price, $p_0 > 0$, in the home country, where p_0 is denominated in the domestic currency. At $t = 1$, after the complete resolution of the exchange rate uncertainty, the MNF has the right, but not the obligation, to acquire additional k_1 units of capital at a known unit price, p_1, in the home country, where p_1 is denominated in the domestic currency and $p_1 > p_0$ to reflect costly expandability of FDI. The MNF's aggregate level of FDI is thus equal to $k_0 + k_1$.

To hedge the exchange rate risk at $t = 0$, the MNF avails itself of customized derivatives at $t = 0$. The payoff of a customized derivative contract at $t = 2$ is delineated by a function, $\phi(e)$, whose functional form is chosen by the MNF at $t = 0$. To focus on the MNF's hedging motive, vis-à-vis its speculative motive, we assume that the contract is fairly priced in that $\mathrm{E}[\phi(\tilde{e})] = 0$, where $\mathrm{E}(\cdot)$ is the expectation operator with respect to $F(e)$. That is, we interpret $\phi(\tilde{e})$ as net of the price of the contract.[4]

Given a realized spot exchange rate, $\tilde{e} = e$, an initial level of FDI, k_0, and a customized derivative contract, $\phi(e)$, the MNF chooses an additional level of FDI, k_1, so as to maximize its domestic currency profit at $t = 2$ under certainty:

$$\max_{k_1 \geq 0} \; ef(k_0 + k_1) - p_0 k_0 - p_1 k_1 - c + \phi(e). \tag{1}$$

[4]If $\mathrm{E}[\phi(\tilde{e})] > (<)\ 0$, the positive (negative) risk premium induces the MNF to speculate by selling (purchasing) the customized derivative contract.

The Kuhn-Tucker condition for program (1) is given by[5]

$$ef'[k_0 + k_1(e, k_0)] - p_1 \leq 0, \tag{2}$$

where $k_1(e, k_0)$ is the solution to program (1). If $e \leq p_1/f'(k_0)$, it follows from condition (2) that $k_1(e, k_0) = 0$. On the other hand, if $e > p_1/f'(k_0)$, condition (2) holds as an equality:

$$ef'[k_0 + k_1(e, k_0)] - p_1 = 0. \tag{3}$$

Since $f''(k) < 0$, it is easily verified that $k_1(e, k_0)$ is strictly increasing in e for all $e > p_1/f'(k_0)$. The flexibility of making sequential FDI decisions thus proffers the MNF a real (call) option to buy additional capital at $t = 1$, which is rationally exercised whenever the realized spot exchange rate is sufficiently favorable, i.e., $e > p_1/f'(k_0)$.

The MNF's random domestic currency profit at $t = 2$ is given by

$$\pi(\tilde{e}) = \tilde{e}f[k_0 + k_1(\tilde{e}, k_0)] - p_0 k_0 - p_1 k_1(\tilde{e}, k_0) - c + \phi(\tilde{e}), \tag{4}$$

where $k_1(e, k_0) = 0$ for all $e \leq p_1/f'(k_0)$ and $k_1(e, k_0)$ is defined in Eq. (3) for all $e > p_1/f'(k_0)$. The MNF is risk averse and possesses a von Neumann-Morgenstern utility function, $u(\pi)$, defined over its domestic currency profit at $t = 2$, π, with $u'(\pi) > 0$ and $u''(\pi) < 0$.[6] The MNF's ex-ante decision problem is to choose an initial level of FDI, k_0, and a customized derivative contract, $\phi(\tilde{e})$, at $t = 0$ so as to maximize the expected utility of its domestic currency profit at $t = 2$:

$$\max_{k_0 \geq 0, \phi(e)} \ \mathrm{E}\{u[\pi(\tilde{e})]\} \quad \text{s.t.} \quad \mathrm{E}[\phi(\tilde{e})] = 0, \tag{5}$$

where $\pi(\tilde{e})$ is defined in Eq. (4).

Figure 1 depicts how the sequence of events unfolds in the model.

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[5]The second-order condition for program (1) is satisfied given the strict concavity of $f(k)$.

[6]The risk-averse behavior of the MNF can be motivated by managerial risk aversion (Stulz, 1984), corporate taxes (Smith and Stulz, 1985), costs of financial distress (Smith and Stulz, 1985), and capital market imperfections (Stulz, 1990; Froot, Scharfstein, and Stein, 1993). See Tufano (1996) for evidence that managerial risk aversion is a rationale for corporate risk management in the gold mining industry.

(Insert Figure 1 here)

3. Optimal FDI and hedging decisions

Given the Inada conditions on $f(k)$ and the fact that $p_1 > p_0$, the solution to program (5) must be an interior one. The first-order conditions for program (5) are given by[7]

$$\int_{\underline{e}}^{p_1/f'(k_0^*)} u'[\pi^*(e)][ef'(k_0^*) - p_0] \, \mathrm{d}F(e) + \int_{p_1/f'(k_0^*)}^{\overline{e}} u'[\pi^*(e)](p_1 - p_0) \, \mathrm{d}F(e) = 0, \quad (6)$$

and

$$u'[\pi^*(e)] - \lambda^* = 0 \quad \text{for all } e \in [\underline{e}, \overline{e}], \quad (7)$$

where Eq. (6) follows from Leibniz's rule and Eq. (3), λ is the Lagrange multiplier, and an asterisk (*) signifies an optimal level. If $p_1 = p_0$, it is evident from Eq. (6) that we have a corner solution to program (5) in that $k_0^* = 0$.

Solving Eqs. (6) and (7) yields our first proposition.

Proposition 1. *If the MNF is allowed to make sequential FDI and to use customized derivatives that are fairly priced for hedging purposes, the MNF's optimal initial level of FDI, k_0^*, solves*

$$\mathrm{E}(\tilde{e})f'(k_0^*) = p_0 + \mathrm{E}\{\max[\tilde{e}f'(k_0^*) - p_1, 0]\}, \quad (8)$$

and the optimal customized derivative contract, $\phi^*(e)$, is given by

$$\phi^*(e) = \nu^* - ef[k_0^* + k_1(e, k_0^*)] + p_1 k_1(e, k_0^*), \quad (9)$$

where $\nu^* = \mathrm{E}\{\tilde{e}f[k_0^* + k_1(\tilde{e}, k_0^*)] - p_1 k_1(\tilde{e}, k_0^*)\}$.

[7]The second-order conditions for program (5) are satisfied given risk aversion and the strict concavity of $f(k)$.

Proof. Using Eq. (4), we can write Eq. (7) as

$$ef[k_0^* + k_1(e, k_0^*)] - p_0 k_0^* - p_1 k_1(e, k_0^*) - c + \phi^*(e) = {u'}^{-1}(\lambda^*) \quad \text{for all } e \in [\underline{e},\ \overline{e}], \tag{10}$$

where ${u'}^{-1}(\lambda^*)$ is a constant. It then follows from $\mathrm{E}[\phi^*(\tilde{e})] = 0$ and Eq. (10) that ${u'}^{-1}(\lambda^*) = \nu^* - p_0 k_0^* - c$, thereby implying equation (9). Substituting equation (7) into equation (6) yields

$$\int_{\underline{e}}^{p_1/f'(k_0^*)} [ef'(k_0^*) - p_0]\ \mathrm{d}F(e) + \int_{p_1/f'(k_0^*)}^{\overline{e}} (p_1 - p_0)\ \mathrm{d}F(e) = 0. \tag{11}$$

Rearranging terms of Eq. (11) yields

$$\int_{\underline{e}}^{\overline{e}} ef'(k_0^*)\ \mathrm{d}F(e) - \int_{p_1/f'(k_0^*)}^{\overline{e}} [ef'(k_0^*) - p_1]\ \mathrm{d}F(e) - p_0 = 0. \tag{12}$$

Eq. (12) is identical to Eq. (8). □

(Insert Figure 2 here)

To see the intuition of Proposition 1, we refer to Figure 2. Using Eq. (4) and setting $\phi(e) \equiv 0$, the MNF's unhedged domestic currency profit at $t = 2$ is given by

$$\hat{\pi}(\tilde{e}) = \tilde{e}f[k_0 + k_1(\tilde{e}, k_0)] - p_0 k_0 - p_1 k_1(\tilde{e}, k_0) - c. \tag{13}$$

If the MNF devises its customized derivative contract to be

$$\phi(e) = \nu - ef[k_0 + k_1(e, k_0)] + p_1 k_1(e, k_0), \tag{14}$$

where $\nu = \mathrm{E}\{\tilde{e}f[k_0 + k_1(\tilde{e}, k_0)] - p_1 k_1(\tilde{e}, k_0)\}$, then its hedged domestic currency profit at $t = 2$ is the sum of Eqs. (13) and (14), i.e., $\pi(\tilde{e}) = \hat{\pi}(\tilde{e}) + \phi(\tilde{e}) = \nu - p_0 k_0 - c$, which is non-stochastic. In this case, the MNF's ex-ante decision problem becomes

$$\max_{k_0 \geq 0} \int_{\underline{e}}^{p_1/f'(k_0)} ef(k_0)\ \mathrm{d}F(e)$$

$$+\int_{p_1/f'(k_0)}^{\overline{e}} \{ef[k_0 + k_1(e, k_0)] - p_1k_1(e, k_0)\} \,\mathrm{d}F(e) - p_0k_0 - c. \tag{15}$$

The solution to program (15) renders Eq. (8). Eq. (8) states that the optimal initial level of FDI, k_0^*, is the one that equates the expected marginal return to FDI made at $t = 0$, $\mathrm{E}(\tilde{e})f'(k_0^*)$, to the unit price of capital at $t = 0$, p_0, plus the forgone option value of waiting to invest that unit of capital at $t = 1$, $\mathrm{E}\{\max[\tilde{e}f'(k_0^*) - p_1, 0]\}$. Substituting k_0^* into Eq. (14) yields Eq. (9).

Two remarks are in order. First, the MNF tailors its optimal customized derivative contract, $\phi^*(e)$, in a way that its hedged domestic currency profit at $t = 2$ is stabilized at the expected level, $\nu^* - p_0k_0^* - c$. The MNF as such faces no risk exposure to $\tilde{e}$. Second, the optimal initial level of FDI, k_0^*, is preference-free. These results resemble the celebrated separation and full-hedging theorems (see, e.g., Broll, 1992; Broll and Zilcha, 1992; Broll, Wong, and Zilcha, 1999; Chang and Wong, 2003; Wong, 2003a) with one caveat: While k_0^* is independent of the MNF's utility function, $u(\pi)$, it does depend on the distribution of the underlying exchange rate uncertainty, $F(e)$, as is evident from Eq. (8).

The optimal customized derivative contract, $\phi^*(e)$, as specified in Eq. (9), takes on a rather complicated form. It is unclear how we can structure this contract in a practical manner. The spanning of $\phi^*(e)$ by a portfolio of plain vanilla derivatives, i.e., currency futures and options that are more readily available, is thus worth examining. We characterize such a replicating portfolio in the following proposition.[8]

Proposition 2. *If the MNF has access to the unbiased currency futures and a continuum of fairly priced currency put and call option contracts of all exercise prices for hedging purposes, the MNF's optimal customized derivative contract, $\phi^*(e)$, can be replicated by trading these plain vanilla derivatives exclusively in the following way:*

$$\phi^*(e) = [e - \mathrm{E}(\tilde{e})]\phi^{*\prime}[\mathrm{E}(\tilde{e})] + \int_{\underline{e}}^{\mathrm{E}(\tilde{e})} [\max(x - e, 0) - v_p(x)]\phi^{*\prime\prime}(x) \,\mathrm{d}x$$

[8]Due to the put-call parity, the replicating portfolio is by no means unique.

$$+\int_{\mathrm{E}(\tilde{e})}^{\overline{e}}[\max(e-x,0)-v_c(x)]\phi^{*\prime\prime}(x)\ \mathrm{d}x, \tag{16}$$

where $v_p(x)=\mathrm{E}[\max(x-\tilde{e},0)]$ *and* $v_c(x)=\mathrm{E}[\max(\tilde{e}-x,0)]$.

Proof. Using the fundamental theorem of calculus, we have

$$\phi^*(e)=\phi^*[\mathrm{E}(\tilde{e})]-I_{\{e<\mathrm{E}(\tilde{e})\}}\int_e^{\mathrm{E}(\tilde{e})}\phi^{*\prime}(y)\ \mathrm{d}y+I_{\{e>\mathrm{E}(\tilde{e})\}}\int_{\mathrm{E}(\tilde{e})}^{e}\phi^{*\prime}(y)\ \mathrm{d}y$$

$$=\phi^*[\mathrm{E}(\tilde{e})]-I_{\{e<\mathrm{E}(\tilde{e})\}}\int_e^{\mathrm{E}(\tilde{e})}\left\{\phi^{*\prime}[\mathrm{E}(\tilde{e})]-\int_y^{\mathrm{E}(\tilde{e})}\phi^{*\prime\prime}(x)\ \mathrm{d}x\right\}\mathrm{d}y$$

$$+I_{\{e>\mathrm{E}(\tilde{e})\}}\int_{\mathrm{E}(\tilde{e})}^{e}\left\{\phi^{*\prime}[\mathrm{E}(\tilde{e})]+\int_{\mathrm{E}(\tilde{e})}^{y}\phi^{*\prime\prime}(x)\ \mathrm{d}x\right\}\mathrm{d}y, \tag{17}$$

where $I_{\{\cdot\}}$ is an indicator function that takes on unity if the event described in the curly brackets occurs, and zero otherwise. Applying Fubini's theorem, we can write Eq. (17) as

$$\phi^*(e)=\phi^*[\mathrm{E}(\tilde{e})]+[e-\mathrm{E}(\tilde{e})]\phi^{*\prime}[\mathrm{E}(\tilde{e})]+I_{\{e<\mathrm{E}(\tilde{e})\}}\int_e^{\mathrm{E}(\tilde{e})}\int_e^{x}\phi^{*\prime\prime}(x)\ \mathrm{d}y\ \mathrm{d}x$$

$$+I_{\{e>\mathrm{E}(\tilde{e})\}}\int_{\mathrm{E}(\tilde{e})}^{e}\int_x^{e}\phi^{*\prime\prime}(x)\ \mathrm{d}y\ \mathrm{d}x. \tag{18}$$

Taking the integral over y in Eq. (18) yields

$$\phi^*(e)=\phi^*[\mathrm{E}(\tilde{e})]+[e-\mathrm{E}(\tilde{e})]\phi^{*\prime}[\mathrm{E}(\tilde{e})]+I_{\{e<\mathrm{E}(\tilde{e})\}}\int_e^{\mathrm{E}(\tilde{e})}(x-e)\phi^{*\prime\prime}(x)\ \mathrm{d}x$$

$$+I_{\{e>\mathrm{E}(\tilde{e})\}}\int_{\mathrm{E}(\tilde{e})}^{e}(e-x)\phi^{*\prime\prime}(x)\ \mathrm{d}y\ \mathrm{d}x$$

$$=\phi^*[\mathrm{E}(\tilde{e})]+[e-\mathrm{E}(\tilde{e})]\phi^{*\prime}[\mathrm{E}(\tilde{e})]+\int_0^{\mathrm{E}(\tilde{e})}\max(x-e,0)\phi^{*\prime\prime}(x)\ \mathrm{d}x$$

$$+\int_{\mathrm{E}(\tilde{e})}^{\infty}\max(e-x,0)\phi^{*\prime\prime}(x)\ \mathrm{d}x. \tag{19}$$

Taking expectations on both side of Eq. (19) with respect to $F(e)$ yields

$$\mathrm{E}[\phi^*(\tilde{e})]=\phi^*[\mathrm{E}(\tilde{e})]+\int_{\underline{e}}^{\mathrm{E}(\tilde{e})}v_p(x)\phi^{*\prime\prime}(x)\ \mathrm{d}x+\int_{\mathrm{E}(\tilde{e})}^{\overline{e}}v_c(x)\phi^{*\prime\prime}(x)\ \mathrm{d}x, \tag{20}$$

where $v_p(x) = \mathrm{E}[\max(x - \tilde{e}, 0)]$ and $v_c(x) = \mathrm{E}[\max(\tilde{e} - x, 0)]$. Substituting Eq. (20) into Eq. (19) and using the fact that $\mathrm{E}[\phi^*(\tilde{e})] = 0$ yields Eq. (16). □

Proposition 2 describes how we can replicate the optimal customized derivative contract, $\phi^*(e)$, as characterized in Eq. (9) by trading the unbiased currency futures and a continuum of fairly priced currency put and call option contracts of all exercise prices. According to Eq. (16), the replicating portfolio consists of buying $\phi^{*\prime}[\mathrm{E}(\tilde{e})]$ units of the futures contracts, $\phi^{*\prime\prime}(x)$ units of the put option contracts with the exercise price, x, for all $x \in [\underline{e}, \mathrm{E}(\tilde{e})]$, and $\phi^{*\prime\prime}(x)$ units of the call option contracts with the exercise price, x, for all $x \in [\mathrm{E}(\tilde{e}), \overline{e}]$. The futures position creates a tangent to the MNF's unhedged domestic currency profit at $\mathrm{E}(\tilde{e})$. The put and call option positions are used to bend the tangent line so as to match the MNF's unhedged domestic currency profit perfectly for all $e \in [\underline{e}, \overline{e}]$. As such, the MNF's hedged domestic currency profit at $t = 2$ is ultimately stabilized at the expected level.[9]

Differentiating Eq. (13) twice with respect to e yields

$$\hat{\pi}''(e) = \begin{cases} 0 & \text{if } e \in [\underline{e}, p_1/f'(k_0)], \\ -f'[k_0 + k_1(e, k_0)]^2/ef''[k_0 + k_1(e, k_0)] & \text{if } e \in (p_1/f'(k_0), \overline{e}], \end{cases} \tag{21}$$

where we have used Eq. (3). It is evident from Eq. (21) that the MNF's unhedged domestic currency profit at $t = 2$ is a convex function of the realized spot exchange rate (see also Figure 2). The MNF's implicit real hedge thus introduces a convex component into its exchange rate risk exposure, thereby calling for the use of currency options for hedging purposes. In the 1998 Wharton survey of financial risk management by US non-financial firms, Bodnar, Hayt, and Marston (1998) report that 68% of the 200 derivatives-using firms indicated that they had used some form of options within the past 12 months. Proposition 2 thus offers a rationale for the hedging demand for currency options by MNFs that make sequential FDI under exchange rate uncertainty.

[9]For any given twice continuously differentiable function of a terminal stock price, Carr and Madan (2001) show how this function can be replicated by positions in pure discount bonds, the underlying stock, and call and put options of all exercise prices. See also Wong (2003b). Proposition 2 is along the line of their results.

4. Lumpy versus sequential FDI

In this section, we consider the case wherein the MNF is unable to adjust its irreversible FDI at $t = 1$ or, equivalently, we set $p_1 = \infty$.

At $t = 0$, the MNF chooses a level of lumpy FDI, k_0, and a customized derivative contract, $\phi(e)$, so as to maximize the expected utility of its domestic currency profit at $t = 2$:

$$\max_{k_0 \geq 0, \phi(e)} \ \mathrm{E}\{u[\tilde{e}f(k_0) - p_0 k_0 - c + \phi(\tilde{e})]\} \quad \text{s.t.} \quad \mathrm{E}[\phi(\tilde{e})] = 0. \tag{22}$$

The first-order conditions for program (22) are given by[10]

$$\mathrm{E}\{u'[\tilde{e}f(k_0^{\mathrm{o}}) - p_0 k_0^{\mathrm{o}} - c + \phi^{\mathrm{o}}(\tilde{e})][\tilde{e}f'(k_0^{\mathrm{o}}) - p_0]\} = 0, \tag{23}$$

and

$$u'[ef(k_0^{\mathrm{o}}) - p_0 k_0^{\mathrm{o}} - c + \phi^{\mathrm{o}}(e)] - \lambda^{\mathrm{o}} = 0 \quad \text{for all } e \in [\underline{e}, \overline{e}], \tag{24}$$

where λ is the Lagrange multiplier, and a nought ($^{\mathrm{o}}$) indicates an optimal level.

Solving Eqs. (23) and (24) yields the following proposition.

Proposition 3. *If the MNF is unable to adjust its irreversible FDI at* $t = 1$ *and is allowed to use customized derivatives that are fairly priced for hedging purposes, the MNF's optimal level of lumpy FDI,* k_0^{o}, *solves*

$$\mathrm{E}(\tilde{e})f'(k_0^{\mathrm{o}}) = p_0, \tag{25}$$

and its optimal customized derivative contract, $\phi^{\mathrm{o}}(e)$, *is given by*

$$\phi^{\mathrm{o}}(e) = [\mathrm{E}(\tilde{e}) - e]f(k_0^{\mathrm{o}}). \tag{26}$$

[10]The second-order conditions for program (22) are satisfied given risk aversion and the strict concavity of $f(k)$.

Proof. It is evident from Eq. (24) that

$$ef(k_0^{\mathrm{o}}) - p_0 k_0^{\mathrm{o}} - c + \phi^{\mathrm{o}}(e) = {u'}^{-1}(\lambda^{\mathrm{o}}) \quad \text{for all } e \in [\underline{e}, \overline{e}], \tag{27}$$

where ${u'}^{-1}(\lambda^{\mathrm{o}})$ is a constant. It then follows from $\mathrm{E}[\phi^{\mathrm{o}}(\tilde{e})] = 0$ and Eq. (27) that ${u'}^{-1}(\lambda^{\mathrm{o}}) = \mathrm{E}(\tilde{p})f(k_0^{\mathrm{o}}) - p_0 k_0^{\mathrm{o}} - c$, thereby implying Eq. (26). Substituting Eq. (24) into Eq. (23) yields Eq. (25). □

The intuition of Proposition 3 is as follows. The MNF could have completely eliminated its exchange rate risk exposure had it chosen $\phi(e) = [\mathrm{E}(\tilde{e}) - e]f(k_0)$, which can be perfectly replicated by a full-hedge via shorting $f(k_0)$ units of the unbiased currency futures contracts at $t = 0$. Alternatively put, the degree of exchange rate risk exposure to be assumed by the MNF should be totally unrelated to its FDI decision at $t = 0$. The optimal level of lumpy FDI, k_0^{o}, is then chosen to maximize $\mathrm{E}(\tilde{e})f(k_0) - p_0 k_0 - c$, thereby yielding Eq. (25). Since the currency futures contracts are unbiased, they offer actuarially fair "insurance" to the MNF. The risk-averse MNF as such optimally opts for full insurance by choosing a full-hedge. These results are simply the well-known separation and full-hedging theorems emanated from the literature on MNFs under exchange rate uncertainty (see, e.g., Broll, 1992; Broll and Zilcha, 1992; Broll, Wong, and Zilcha, 1999; Chang and Wong, 2003; Wong, 2003a).

Since $\max[\tilde{e}f'(k_0^*) - p_1, 0] \geq 0$, Eq. (8) implies that $\mathrm{E}(\tilde{e})f'(k_0^*) > p_0$. It then follows from $f''(k) < 0$ and Eq. (25) that $k_0^* < k_0^{\mathrm{o}}$, thereby invoking the following proposition.

Proposition 4. *If the MNF is allowed to use customized derivatives that are fairly priced for hedging purposes, the MNF's optimal initial level of FDI in the case of sequential FDI is less than that in the case of lumpy FDI, i.e., ,* $k_0^* < k_0^{\mathrm{o}}$.

The intuition of Proposition 4 is as follows. The flexibility of making sequential FDI, vis-à-vis lumpy FDI, offers the MNF a real (call) option to buy additional capital at $t = 1$, which is exercised whenever $e > p_1/f'(k_0)$. It is well-known that the value of a call option increases with a decrease in its exercise price (Merton, 1973). Since $f''(k) < 0$, the MNF has incentives to cut down its initial level of FDI, k_0, so as to lower $p_1/f'(k_0)$, the exercise price of the real option created by the flexibility of making sequential FDI. Thus, we have $k_0^* < k_0^{\rm o}$.

Proposition 4 states that the MNF is forced to undertake more FDI at $t = 0$ should FDI be lumpy rather than sequential. It is of interest to extend this result by comparing the MNF's expected optimal aggregate level of sequential FDI, $k_0^* + \mathrm{E}[k_1(\tilde{e}, k_0^*)]$, with that of lumpy FDI, $k_0^{\rm o}$. Since $k_0^* < k_0^{\rm o}$ and $\mathrm{E}[k_1(\tilde{e}, k_0^*)] > 0$, such a comparison is a non-trivial one. To see this, consider the extreme case wherein $p_1 = p_0$. In this case, we know from Eq. (6) that $k_0^* = 0$. It then follows from Eqs. (3) and (25) that $k_1[\mathrm{E}(\tilde{e}), 0] = k_0^{\rm o}$. Totally differentiating $ef'[k_1(e, 0)] = p_0$ with respect to e twice and rearranging terms yields

$$\frac{\partial^2 k_1(e,0)}{\partial e^2} = -\frac{f'[k_1(e,0)]}{e^2 f''[k_1(e,0)]}\left\{\frac{f'[k_1(e,0)]f'''[k_1(e,0)]}{f''[k_1(e,0)]^2} - 2\right\}. \tag{28}$$

It follows from Eq. (28) that $k_1(e, 0)$ is convex (concave) in e if $f'(k)f'''(k)/f''(k)^2$ is everywhere no less (no greater) than 2. By Jensen's inequality, the convexity (concavity) of $k_1(e, 0)$ in e implies that $\mathrm{E}[k_1(\tilde{e}, 0)] > (<)\ k_1[\mathrm{E}(\tilde{e}), 0] = k_0^{\rm o}$. We thus establish the following proposition.

Proposition 5. *Given that $p_1 = p_0$, the MNF's expected optimal aggregate level of sequential FDI,* $\mathrm{E}[k_1(\tilde{e}, 0)]$*, is greater or smaller than the optimal level of lumpy FDI, $k_0^{\rm o}$, depending on whether $f'(k)f'''(k)/f''(k)^2$ is everywhere no less or no greater than 2, respectively.*

If $f'''(k) \leq 0$, then $f'(k)f'''(k)/f''(k)^2 \leq 0$ so that $k_1(e, 0)$ is concave in e. In this case, we have $\mathrm{E}[k_1(\tilde{e}, 0)] < k_0^{\rm o}$. On the other hand, if $f(k) = k^\alpha$, where $0 < \alpha < 1$, then $f'(k)f'''(k)/f''(k)^2 = 2 + \alpha/(1-\alpha) > 2$ so that $k_1(e, 0)$ is convex in e. In this case, we have $\mathrm{E}[k_1(\tilde{e}, 0)] > k_0^{\rm o}$. In general, without knowing the specific functional forms of $u(\pi)$, $f(k)$, and $F(e)$, we are a priori unable to make an unambiguous comparison between the MNF's expected optimal aggregate level of sequential FDI and the optimal level of lumpy FDI.

5. Sequential FDI without currency hedging

In this section, we consider the case wherein the MNF is banned from engaging in currency hedging. This is tantamount to setting $\phi(e) \equiv 0$.

At $t = 0$, the MNF chooses a level of FDI, k_0, so as to maximize the expected utility of its domestic currency profit at $t = 2$:

$$\max_{k_0} \ \mathrm{E}\{u[\hat{\pi}(\tilde{e})]\}, \tag{29}$$

where $\hat{\pi}(\tilde{e})$ is defined in Eq. (13). The first-order condition for program (29) is given by[11]

$$\int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u'[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0] \, \mathrm{d}F(e) + \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} u'[\hat{\pi}^\diamond(e)](p_1 - p_0) \, \mathrm{d}F(e) = 0, \tag{30}$$

where a diamond ($^\diamond$) indicates an optimal level.

Rearranging terms of Eq. (30) yields

$$\mathrm{E}\bigg\{\frac{u'[\hat{\pi}^\diamond(\tilde{e})]}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}}\tilde{e}\bigg\} f'(k_0^\diamond) = p_0 + \mathrm{E}\bigg\{\frac{u'[\hat{\pi}^\diamond(\tilde{e})]}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}} \max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\bigg\}. \tag{31}$$

Define the following function:

$$G(e) = \int_{\underline{e}}^{e} \frac{u'[\hat{\pi}^\diamond(x)]}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}} \, \mathrm{d}F(x), \tag{32}$$

for all $e \in [\underline{e},\ \overline{e}]$. It is evident from Eq. (32) that $G'(e) > 0$, $G(\underline{e}) = 0$, and $G(\overline{e}) = 1$. We can as such interpret $G(e)$ as a cumulative distribution function of $\tilde{e}$. Substituting Eq. (32) into Eq. (31) yields

$$\mathrm{E}_G(\tilde{e})f'(k_0^\diamond) = p_0 + \mathrm{E}_G\{\max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\}, \tag{33}$$

where $\mathrm{E}_G(\cdot)$ is the expectation operator with respect to $G(e)$. Eq. (33) states that the optimal initial level of FDI, $k_0^\diamond$, is the one that equates the expected marginal return to FDI made at $t = 0$, $\mathrm{E}_G(\tilde{e})f'(k_0^\diamond)$, to the unit price of capital at $t = 0$, p_0, plus the foregone

[11] The second-order condition for program (29) is satisfied given risk aversion and the strict concavity of $f(k)$.

option value of waiting to invest that unit of capital at $t = 1$, $\mathrm{E}_G\{\max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\}$, where the expectations are evaluated taking the MNF's risk attitude into account.

Using the covariance operator with respect to $F(e)$, $\mathrm{Cov}(\cdot, \cdot)$, we can write Eq. (31) as[12]

$$\left\{\mathrm{E}(\tilde{e}) + \frac{\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \tilde{e}\}}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}}\right\} f'(k_0^\diamond)$$

$$= p_0 + \left\{\mathrm{E}\{\max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\} + \frac{\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\}}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}}\right\}. \tag{34}$$

From Eqs. (3) and (13), we know that $\hat{\pi}^{\diamond\prime}(e) = f[k_0^\diamond + k_1(e, k_0^\diamond)] > 0$. Since $u''(\pi) < 0$, we have $\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \tilde{e}\} < 0$ and $\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\} < 0$. Inspection of Eqs. (8), (33), and (34) reveals that the absence of currency hedging reduces both the expected marginal return to FDI made at $t = 0$ and the forgone option value of waiting to invest that unit of capital at $t = 1$, as compared to those in the presence of currency hedging. The former has a negative effect on the MNF's initial level of FDI while the latter has a positive effect. We show in the following proposition that the net effect is unambiguously negative.

Proposition 6. *Suppose that the MNF is banned from engaging in currency hedging. The MNF's ex-ante and ex-post incentives to make FDI are reduced as compared to those with perfect currency hedging, i.e.,* $k_0^\diamond < k_0^*$ *and* $k_0^\diamond + k_1(e, k_0^\diamond) \leq k_0^* + k_1(e, k_0^*)$, *where the inequality is strict for all* $e < p_1/f'(k_0^*)$.

Proof. Using the fact that $\tilde{e}f'(k_0^\diamond) - p_1 = \max[\tilde{e}f'(k_0^\diamond) - p_1, 0] - \max[p_1 - \tilde{e}f'(k_0^\diamond), 0]$, we can write Eq. (34) as

$$\mathrm{E}(\tilde{e})f'(k_0^\diamond) = p_0 + \mathrm{E}\{\max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\} + \frac{\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \max[p_1 - \tilde{e}f'(k_0^\diamond), 0]\}}{\mathrm{E}\{u'[\hat{\pi}^\diamond(\tilde{e})]\}}. \tag{35}$$

Since $\mathrm{Cov}\{u'[\hat{\pi}^\diamond(\tilde{e})], \max[p_1 - \tilde{e}f'(k_0^\diamond), 0]\} > 0$, Eq. (35) implies that $\mathrm{E}(\tilde{e})f'(k_0^\diamond) > p_0 + \mathrm{E}\{\max[\tilde{e}f'(k_0^\diamond) - p_1, 0]\}$. It then follows from Eq. (8) and the strict concavity of $f(k)$ that $k_0^\diamond < k_0^*$. From Eq. (3), we have $k_0^\diamond + k_1(e, k_0^\diamond) < k_0^*$ for all $e < p_1/f'(k_0^*)$ and $k_0^\diamond + k_1(e, k_0^\diamond) = k_0^* + k_1(e, k_0^*)$ for all $e \geq p_1/f'(k_0^*)$. $\square$

[12]For any two random variables, $\tilde{x}$ and $\tilde{y}$, we have $\mathrm{Cov}(\tilde{x}, \tilde{y}) = \mathrm{E}(\tilde{x}\tilde{y}) - \mathrm{E}(\tilde{x})\mathrm{E}(\tilde{y})$.

To see the intuition underlying Proposition 6, we partially differentiate Eq. (13) with respect to k_0 to yield

$$\frac{\partial \hat{\pi}(e)}{\partial k_0} = ef'[k_0 + k_1(e, k_0)] - p_0, \tag{36}$$

which is negative for all $e < p_0/f'(k_0)$ and positive for all $e > p_0/f'(k_0)$. If the MNF is allowed to use customized derivatives for hedging purposes, the optimal initial level of FDI is k_0^*. When the MNF is banned from engaging in currency hedging, risk aversion implies that the MNF has incentives to shift its domestic currency profits when the realizations of $\tilde{e}$ are high to those when the realizations of $\tilde{e}$ are low. This can be achieved by lowering k_0, as is evident from Eq. (36). Hence, we must have $k_0^\diamond < k_0^*$. It then follows from Eq. (3) that $k_0^\diamond + k_1(e, k_0^\diamond) < k_0^*$ for all $e < p_1/f'(k_0^*)$ and $k_0^\diamond + k_1(e, k_0^\diamond) = k_0^* + k_1(e, k_0^*)$ for all $e \geq p_1/f'(k_0^*)$. Thus, both the optimal initial level of FDI and the optimal aggregate level of FDI are lower in the absence than in the presence of currency hedging.

Unlike k_0^*, the optimal initial level of FDI in the absence of currency hedging, $k_0^\diamond$, is not preference-free, as is evident from Eq. (33). It is of interest to examine how the MNF in this case alters its FDI decisions when it becomes more risk averse. To this end, we let $v(\pi)$ be a von Neumann-Morgenstern utility function that is more risk averse than $u(\pi)$. According to Pratt (1964), we can write $v(\pi) = \psi[u(\pi)]$, where $\psi(\cdot)$ is a strictly concave function.

The more risk-averse MNF's ex-ante decision problem is given by

$$\max_{k_0 \geq 0} \ \mathrm{E}\{v[\hat{\pi}(\tilde{e})]\}, \tag{37}$$

where $\hat{\pi}(\tilde{e})$ is defined in Eq. (13). Differentiating the objective function in program (37) with respect to k_0, and evaluating the resulting derivative at $k_0 = k_0^\diamond$ yields

$$\begin{aligned}\left.\frac{\mathrm{dE}\{v[\hat{\pi}(\tilde{e})]\}}{\mathrm{d}k_0}\right|_{k_0 = k_0^\diamond} &= \int_{\underline{e}}^{p_1/f'(k_0^\diamond)} \psi'\{u[\hat{\pi}^\diamond(e)]\}u'[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0] \ \mathrm{d}F(e) \\ &\quad + \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} \psi'\{u[\hat{\pi}^\diamond(e)]\}u'[\hat{\pi}^\diamond(e)](p_1 - p_0) \ \mathrm{d}F(e).\end{aligned} \tag{38}$$

Multiplying $\psi'\{u\{\hat{\pi}^\diamond[p_0/f'(k_0^\diamond)]\}\}$ to Eq. (30) and substituting the resulting equation to the right-hand side of Eq. (38) yields

$$\int_{\underline{e}}^{p_1/f'(k_0^\diamond)} \Big\{\psi'\{u[\hat{\pi}^\diamond(e)]\} - \psi'\{u\{\hat{\pi}^\diamond[p_0/f'(k_0^\diamond)]\}\}\Big\} u'[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]\,\mathrm{d}F(e)$$

$$+\int_{p_1/f'(k_0^\diamond)}^{\overline{e}} \Big\{\psi'\{u[\hat{\pi}^\diamond(e)]\} - \psi'\{u\{\hat{\pi}^\diamond[p_0/f'(k_0^\diamond)]\}\}\Big\} u'[\hat{\pi}^\diamond(e)](p_1 - p_0)\,\mathrm{d}F(e).$$

The above expression is unambiguously negative because the strict concavity of $\psi(\cdot)$ implies that $\psi'\{u[\hat{\pi}^\diamond(e)]\} > (<)\ \psi'\{u\{\hat{\pi}^\diamond[p_0/f'(k_0^\diamond)]\}\}$ for all $e < (>)\ p_0/f'(k_0^\diamond)$. Thus, the MNF must invest less than $k_0^\diamond$ at $t = 0$ when it becomes more risk averse, thereby invoking the following proposition.

Proposition 7. *Suppose that the MNF is banned from engaging in currency hedging. The perverse ex-ante and ex-post incentives to make FDI, as characterized in Proposition 6, are preserved in the case of increased risk aversion from $u(\pi)$ to $v(\pi)$.*

Finally, we want to examine how the fixed cost, c, for the access to the project would affect the MNF's ex-ante and ex-post incentives to make FDI. As is well known in the literature on decision making under certainty, risk aversion alone is usually too weak to yield intuitively appealing comparative statics. To reconcile these shortcomings, the literature suggests that it is reasonable and useful to impose the additional assumption of decreasing absolute risk aversion (Gollier, 2001). We say that the MNF's utility function, $u(\pi)$, exhibits decreasing absolute risk aversion if, and only if, its Arrow-Pratt measure of absolute risk aversion, $-u''(\pi)/u'(\pi)$, decreases with π.[13] We state and prove the following proposition.

Proposition 8. *Suppose that the MNF is banned from engaging in currency hedging. If the MNF's utility function exhibits decreasing absolute risk aversion, an increase in the fixed cost for the access to the project reduces the MNF's ex-ante and ex-post incentives to make FDI.*

[13] If the MNF's utility function satisfies increasing (constant) absolute risk aversion, it can be shown analogously that $\mathrm{d}k_0^\diamond/\mathrm{d}c > (=)\ 0$.

Proof. Totally differentiating Eq. (30) with respect to c and rearranging terms yields

$$\frac{\mathrm{d}k_0^\diamond}{\mathrm{d}c} = \frac{1}{\Delta}\bigg\{\int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u''[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]\ \mathrm{d}F(e)$$

$$+\int_{p_1/f'(k_0^\diamond)}^{\overline{e}} u''[\hat{\pi}^\diamond(e)](p_1 - p_0)\ \mathrm{d}F(e)\bigg\}, \tag{39}$$

where $\Delta = \int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u''[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]^2\ \mathrm{d}F(e) + \int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u'[\hat{\pi}^\diamond(e)]ef''(k_0^\diamond)\ \mathrm{d}F(e) + \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} u''[\hat{\pi}^\diamond(e)](p_1 - p_0)^2\ \mathrm{d}F(e) < 0$. From Eqs. (3) and (13), we have $\hat{\pi}^{\diamond\prime}(e) = f[k_0^\diamond + k_1(e, k_0^\diamond)] > 0$. Since $u(\pi)$ satisfies decreasing absolute risk aversion, we have

$$-\frac{u''[\hat{\pi}^\diamond(e)]}{u'[\hat{\pi}^\diamond(e)]} > (<)\ R \quad \text{for all} \quad e < (>)\ p_0/f'(k_0^\diamond), \tag{40}$$

where R is the Arrow-Pratt measure of absolute risk aversion evaluated at $e = p_0/f'(k_0^\diamond)$. We multiply $-u'[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]$ to both sides of inequality (40) for all $e < p_0/f'(k_0^\diamond)$, and $-u'[\hat{\pi}^\diamond(e)](p_1 - p_0)$ to both sides of inequality (40) for all $e > p_0/f'(k_0^\diamond)$. Taking the expectations on both sides of the resulting inequality with respect to $F(e)$ yields

$$\int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u''[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]\ \mathrm{d}F(e) + \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} u''[\hat{\pi}^\diamond(e)](p_1 - p_0)\ \mathrm{d}F(e)$$

$$> -R\bigg\{\int_{\underline{e}}^{p_1/f'(k_0^\diamond)} u'[\hat{\pi}^\diamond(e)][ef'(k_0^\diamond) - p_0]\ \mathrm{d}F(e) + \int_{p_1/f'(k_0^\diamond)}^{\overline{e}} u'[\hat{\pi}^\diamond(e)](p_1 - p_0)\ \mathrm{d}F(e)\bigg\}$$

$$= 0, \tag{41}$$

where the equality follows from Eq. (30). Hence, Eq. (39) and inequality (41) imply that $\mathrm{d}k_0^\diamond/\mathrm{d}c < 0$. □

The intuition of Proposition 8 is as follows. An increase in the fixed cost, c, reduces the MNF's domestic currency profit at $t = 2$ by the same amount for all $e \in [\underline{e}, \overline{e}]$. Given decreasing absolute risk aversion, the MNF becomes more risk averse. It then follows from Proposition 7 that the MNF's ex-ante and ex-post incentives to make FDI are reduced.

Changes in fixed or setup costs incurred by MNFs may be due to changes in investment tax credit offered by the host government, or due to changes in the severity of entry barriers in the host country. Proposition 8 thus implies that FDI flows are positively related to higher investment tax credits and negatively related to more barriers to entry. Hines (2001) finds that the volume of Japanese FDI in countries with whom Japan has "tax sparing" agreements is 1.4 to 2.4 times higher than it would have been otherwise. Furthermore, Anand and Kogut (1997) document that industrial concentration has a negative effect on the attraction of FDI. The implications of Proposition 8 are largely consistent with these empirical findings.

6. Sequential FDI with futures hedging only

In this section, we consider the case wherein the MNF is restricted to use the unbiased currency futures contracts as the sole hedging instrument. This is tantamount to setting $\phi(e) \equiv [\mathrm{E}(\tilde{e}) - e]h$, where h is the number of the currency futures contracts sold (purchased if negative) by the MNF at $t = 0$.

The MNF's random domestic currency profit at $t = 2$ is given by

$$\pi(\tilde{e}) = \tilde{e}f[k_0 + k_1(\tilde{e}, k_0)] - p_0k_0 - p_1k_1(\tilde{e}, k_0) - c + [\mathrm{E}(\tilde{e}) - \tilde{e}]h, \tag{42}$$

where $[\mathrm{E}(\tilde{e}) - e]h$ is the gain or loss from its futures position, h. The MNF's ex-ante decision problem is to choose an initial level of FDI, k_0, and a futures position, h, at $t = 0$ so as to maximize the expected utility of its domestic currency profit at $t = 2$:

$$\max_{k_0 \geq 0, h} \ \mathrm{E}\{u[\pi(\tilde{e})]\}, \tag{43}$$

where $\pi(\tilde{e})$ is defined in Eq. (42). The first-order conditions for program (43) are given by[14]

$$\int_{\underline{e}}^{p_1/f'(k_0^{**})} u'[\pi^{**}(e)][ef'(k_0^{**})-p_0]\,\mathrm{d}F(e)+\int_{p_1/f'(k_0^{**})}^{\overline{e}} u'[\pi^{**}(e)](p_1-p_0)\,\mathrm{d}F(e)=0,\tag{44}$$

and

$$\mathrm{E}\{u'[\pi^{**}(\tilde{e})][\mathrm{E}(\tilde{e})-\tilde{e}]\}=0,\tag{45}$$

where Eq. (44) follows from Leibniz's rule and Eq. (3), and a double asterisk (**) signifies an optimal level.

Solving Eqs. (44) and (45) yields our first proposition.

Proposition 9. *Suppose that the MNF is restricted to use the unbiased currency futures contracts as the sole hedging instrument. The MNF's optimal initial level of FDI in the case of sequential FDI is less than that in the case of lumpy FDI, i.e.,* $k_0^{**}<k_0^{\mathrm{o}}$. *Furthermore, the MNF optimally opts for an over-hedge, i.e.,* $h^{**}>f(k_0^{**})$, *in the case of sequential FDI and a full-hedge, i.e.,* $h^{\mathrm{o}}=f(k_0^{\mathrm{o}})$, *in the case of lumpy FDI.*

Proof. From Proposition 3, we know that the MNF optimally opts for a full-hedge via the unbiased currency futures contracts in the case of lumpy FDI, even when the MNF is allowed to use customized derivatives that are fairly priced for hedging purposes. Thus, the results in Proposition 3 hold when we restrict the MNF to use the unbiased currency futures contracts as the sole hedging instrument.

Rearranging terms of Eq. (44) yields

$$\mathrm{E}\{u'[\pi^{**}(\tilde{e})][\tilde{e}f'(k_0^{**})-p_0]\}-\mathrm{E}\{u'[\pi^{**}(\tilde{e})]\max[\tilde{e}f'(k_0^{**})-p_1,0]\}=0.\tag{46}$$

Multiplying $f'(k_0^{**})$ to Eq. (45) and adding the resulting equation to Eq. (46) yields

$$\mathrm{E}(\tilde{e})f'(k_0^{**})=p_0+\mathrm{E}\left\{\frac{u'[\pi^{**}(\tilde{e})]}{\mathrm{E}\{u'[\pi^{**}(\tilde{e})]\}}\max[\tilde{e}f'(k_0^{**})-p_1,0]\right\}>p_0,\tag{47}$$

[14]The second-order conditions for program (43) are satisfied given risk aversion and the strict concavity of $f(k)$.

where the inequality follows from $u'(\pi) > 0$ and $\max[\tilde{e}f'(k_0^{**}) - p_1, 0] \geq 0$. Since $f''(k) < 0$, Eqs. (25) and (47) imply that $k_0^{**} < k_0^{\mathrm{o}}$.

Using the covariance operator with respect to $F(e)$, we can write Eq. (45) as

$$\mathrm{Cov}\{u'[\pi^{**}(\tilde{e})], \tilde{e}\} = 0. \tag{48}$$

Partially differentiating $u'[\pi^{**}(e)]$ with respect to e yields

$$\frac{\partial}{\partial e} u'[\pi^{**}(e)] = u''[\pi^{**}(e)]\{f[k_0^{**} + k_1(e, k_0^{**})] - h^{**}\}, \tag{49}$$

where we have used Eq. (3). Suppose that $h^{**} \leq f(k_0^{**})$. Since $k_1(e, k_0^{**}) \geq 0$ and $f'(k) > 0$, Eq. (49) implies that $\mathrm{Cov}\{u'[\pi^{**}(\tilde{e})], \tilde{e}\} < 0$, which contradicts Eq. (48). Thus, it must be true that $h^{**} > f(k_0^{**})$. $\square$

The intuition of Proposition 9 is as follows. In the case of sequential FDI, the MNF is endowed with a real (call) option to buy additional capital at $t = 1$, which is exercised whenever $e > p_1/f'(k_0)$. The value of such a call option increases with a decrease in its exercise price (Merton, 1973) so that the MNF has incentives to cut down its initial level of FDI, k_0, thereby implying that $k_0^{**} < k_0^{\mathrm{o}}$. Given the fact that its exchange rate risk exposure is at least $f(k_0^{**})$ and is strictly greater than $f(k_0^{**})$ when $e > p_1/f'(k_0^{**})$, the MNF opts for $h^{**} > f(k_0^{**})$ as its optimal futures position. Unlike in the case of lumpy FDI, the MNF has to bear some residual exchange rate risk that cannot be eliminated by trading the unbiased currency futures contracts only.

Using a sample of U.S. MNFs, Allayannis, Ihrig, and Weston (2001) find that operational hedging is not an effective substitute for financial hedging. In spite of this, operational hedging is capable of reducing exchange rate risk exposure and enhancing firm value when it is used in combination with financial hedging. This is confirmed by Pantzalis, Simkins, and Laux (2001) and Kim, Mathur, and Nam (2006) who also show that operational hedging serves as a real option for managing exchange rate risk. Furthermore, Allayannis, Ihrig, and Weston (2001) and Kim, Mathur, and Nam (2006) find that operational hedging is a complement to, not a substitute for, financial hedging. Since a full-hedge is the optimal futures position in the case of lumpy FDI, the over-hedging result of Proposition 9 when the MNF is allowed to make sequential FDI is consistent with the empirical finding that operational hedges and financial hedges are complements to each other.

Define the following function:

$$H(e)=\int_{\underline{e}}^{e}\frac{u'[\pi^{**}(x)]}{\mathrm{E}\{u'[\pi^{**}(\tilde{e})]\}}\,\mathrm{d}F(x), \tag{50}$$

for all $e\in[\underline{e},\ \overline{e}]$. It is evident from Eq. (50) that $H'(e)>0$, $H(\underline{e})=0$, and $H(\overline{e})=1$. We can as such interpret $H(e)$ as a cumulative distribution function of $\tilde{e}$. Substituting Eq. (50) into Eq. (47) yields

$$\mathrm{E}(\tilde{e})f'(k_0^{**})=p_0+\mathrm{E}_H\{\max[\tilde{e}f'(k_0^{**})-p_1,0]\}, \tag{51}$$

where $\mathrm{E}_H(\cdot)$ is the expectation operator with respect to $H(e)$. The interpretation of Eq. (51) is similar to those of Eqs. (8) and (33) with two caveats. First, when the unbiased currency futures contracts are the sole hedging instrument, the MNF's risk preferences play no role in determining the expected return to FDI made at $t=0$, which is governed solely by $F(e)$, as is evident from the left-hand side of Eq. (51). This is simply the spanning property that arises from the tradability of $\tilde{e}$ via the currency futures contracts. Second, the option value of waiting to make FDI at $t=1$ is now priced based on $H(e)$, as is evident from the right-hand side of Eq. (51). Using the covariance operator with respect to $F(e)$, we can write this option value as

$$\begin{aligned}\mathrm{E}_H\{\max[\tilde{e}f'(k_0^{**})-p_1,0]\}&=\mathrm{E}\{\max[\tilde{e}f'(k_0^{**})-p_1,0]\}\\&\quad+\frac{\mathrm{Cov}\{u'[\pi^{**}(\tilde{e})],\max[\tilde{e}f'(k_0^{**})-p_1,0]\}}{\mathrm{E}\{u'[\pi^{**}(\tilde{e})]\}}.\end{aligned} \tag{52}$$

The wedge between this option value and the option value under perfect currency hedging is gauged by the covariance term on the right-hand side of Eq. (52). Due to the non-tradability of the real option embedded in sequential FDI, spanning is not possible and thus the MNF's risk preferences affect the pricing of the option in this incomplete market context.

Partially differentiating $\pi^{**}(e)$ with respect to e yields

$$\pi^{**'}(e) = f[k_0^{**} + k_1(e, k_0^{**})] - h^{**}, \tag{53}$$

where we have used Eq. (3). From Proposition 9, we know that $h^{**} > f(k_0^{**})$. Eq. (53) then implies that $\pi^{**}(e)$ is strictly decreasing for all $e < e_0$ and strictly increasing for all $e > e_0$, where e_0 solves $f[k_0^{**} + k_1(e_0, k_0^{**})] = h^{**}$. Since $\pi^{**}(e)$ is non-monotonic in e, the sign of the covariance term on the right-hand side of Eq. (52) is not immediately determinate. We prove in the following proposition that this term is unambiguously negative.

Proposition 10. *Suppose that the MNF is restricted to use the unbiased currency futures contracts as the sole hedging instrument. The MNF's ex-ante and ex-post incentives to make FDI are enhanced as compared to those with perfect currency hedging, i.e.,* $k_0^{**} > k_0^*$ *and* $k_0^{**} + k_1(e, k_0^{**}) \geq k_0^* + k_1(e, k_0^*)$, *where the inequality is strict for all* $e < p_1/f'(k_0^{**})$.

Proof. From Proposition 9, we know that $h^{**} > f(k_0^{**})$. Eq. (49) then implies that $u'[\pi^{**}(e)]$ is strictly increasing for all $e < e_0$ and strictly decreasing for all $e > e_0$, where e_0 solves $f[k_0^{**}, k_1(e_0, k_1^{**})] = h^{**}$. In other words, $u'[\pi^{**}(e)]$ is hump-shaped and attains a unique global maximum at $e = e_0$. Since $\mathrm{E}\{u'[\pi^{**}(\tilde{e})]\}$ is the expected value of $u'[\pi^{**}(\tilde{e})]$, there must exist at least one and at most two distinct points at which $u'[\pi^{**}(e)] = \mathrm{E}\{u'[\pi^{**}(\tilde{e})]\}$. Write Eq. (45) as

$$\int_{\underline{e}}^{\overline{e}} \Big\{ u'[\pi^{**}(e)] - \mathrm{E}\{u'[\pi^{**}(\tilde{e})]\} \Big\} (e - y) \, \mathrm{d}F(e) = 0, \tag{54}$$

for all $y \in [\underline{e}, \overline{e}]$. If there is only one point, $\hat{e}$, at which $u'[\pi^{**}(\hat{e})] = \mathrm{E}\{u'[\pi^{**}(\tilde{e})]\}$, then we have

$$\int_{\underline{e}}^{\overline{e}} \Big\{ u'[\pi^{**}(e)] - \mathrm{E}\{u'[\pi^{**}(\tilde{e})]\} \Big\} (e - \hat{e}) \, \mathrm{d}F(e) > (<)\ 0, \tag{55}$$

when $u'[\pi^{**}(\underline{e})] \leq (>)\ \mathrm{E}\{u'[\pi^{**}(\tilde{e})]\}$, a contradiction to Eq. (54). Thus, there must exist two distinct points, e_1 and e_2, with $\underline{e} < e_1 < e_0 < e_2 < \overline{e}$, such that $u'[\pi^{**}(e)] \geq \mathrm{E}\{u'[\pi^{**}(\tilde{e})]\}$ for all $e \in [e_1, e_2]$ and $u'[\pi^{**}(e)] < \mathrm{E}\{u'[\pi^{**}(\tilde{e})]\}$ for all $e \in [\underline{e}, e_1) \bigcup (e_2, \overline{e}]$, where the equality holds only at $e = e_1$ and $e = e_2$.

Consider the following function:

$$\begin{aligned} g(x) &= \mathrm{Cov}\{u'[\pi^{**}(\tilde{e})], \max[\tilde{e} - x, 0]\} \\ &= \int_x^{\overline{e}} \Big\{ u'[\pi^{**}(e)] - \mathrm{E}\{u'[\pi^{**}(\tilde{e})]\} \Big\} (e - x)\ \mathrm{d}F(e). \end{aligned} \tag{56}$$

Differentiating Eq. (56) with respect to x and using Leibniz's rule yields

$$g'(x) = -\int_x^{\overline{e}} \Big\{ u'[\pi^{**}(e)] - \mathrm{E}\{u'[\pi^{**}(\tilde{e})]\} \Big\}\ \mathrm{d}F(e). \tag{57}$$

Differentiating Eq. (57) with respect to x and using Leibniz's rule yields

$$g''(x) = \Big\{ u'[\pi^{**}(x)] - \mathrm{E}\{u'[\pi^{**}(\tilde{e})]\} \Big\} F'(x). \tag{58}$$

It follows from Eq. (58) that $g''(x) \geq 0$ for all $x \in [e_1, e_2]$ and $g''(x) < 0$ for all $x \in [\underline{e}, e_1) \bigcup (e_2, \overline{e}]$, where the equality holds only at $x = e_1$ and $x = e_2$. In words, $g(x)$ is strictly concave for all $x \in [\underline{e}, e_1) \bigcup (e_2, \overline{e}]$ and is strictly convex for all $x \in (e_1, e_2)$. It follows from Eq. (57) that $g'(\underline{e}) = g'(\overline{e}) = 0$. Hence, $g(x)$ attains two local maxima at $x = \underline{e}$ and $x = \overline{e}$. From Eq. (56), we have $g(\overline{e}) = 0$. Also, Eqs. (54) and (56) imply that

$$g(\underline{e}) = \int_{\underline{e}}^{\overline{e}} \Big\{ u'[\pi^{**}(e)] - \mathrm{E}\{u'[\pi^{**}(\tilde{e})]\} \Big\} (e - \underline{e})\ \mathrm{d}F(e) = 0.$$

In words, $g(x)$ has an inverted bell-shape bounded from above by zero at $x = \underline{e}$ and $x = \overline{e}$. Hence, $g(x) < 0$ for all $x \in (\underline{e}, \overline{e})$.

In particular, we have $g[p_1/f'(k_0^{**})] < 0$ and thus $\mathrm{Cov}\{u'[\pi^{**}(\tilde{e})], \max[\tilde{e}f'(k_0^{**}) - p_1, 0]\} < 0$. Eq. (52) implies that $\mathrm{E}(\tilde{e})f'(k_0^{**}) < p_0 + \mathrm{E}\{\max[\tilde{e}f'(k_0^{**}) - p_1, 0]\}$. It then follows from Eq. (8) and the strict concavity of $f(k)$ that $k_0^{**} > k_0^*$. From Eq. (3), we have $k_0^{**} > k_0^* + k_1(e, k_0^*)$ for all $e < p_1/f'(k_0^{**})$ and $k_0^{**} + k_1(e, k_0^{**}) = k_0^* + k_1(e, k_0^*)$ for all $e \geq p_1/f'(k_0^{**})$. □

The results of Proposition 10 should be contrasted with those of Proposition 6. If there are only the unbiased currency futures contracts available to the MNF for hedging purposes, risk aversion has no effect on the expected marginal return to FDI made at $t = 0$ but has a negative effect on the forgone option value of waiting to invest that unit of capital at $t = 1$, as is evident from Eq. (52). Thus, the MNF is induced to make more FDI at $t = 0$ as compared to the case of perfect currency hedging, thereby implying that $k_0^{**} > k_0^*$. It then follows from Eq. (3) that $k_0^{**} > k_0^* + k_1(e, k_0^*)$ for all $e < p_1/f'(k_0^{**})$ and $k_0^{**} + k_1(e, k_0^{**}) = k_0^* + k_1(e, k_0^*)$ for all $e \geq p_1/f'(k_0^{**})$. Thus, both the optimal initial level of FDI and the optimal aggregate level of FDI are higher when the MNF is restricted to use the unbiased currency futures contracts as the sole hedging instrument. The opposite results, however, hold when the MNF is banned from engaging in currency hedging (see Proposition 6). An immediate implication is that futures hedging promotes FDI, both ex ante and ex post, a result in line with the extant literature on lumpy FDI (see, e.g., Broll, 1992; Broll and Zilcha, 1992; Broll, Wong, and Zilcha, 1999; Wong, 2003b). This is consistent with the complementary nature of operational and financial hedging strategies as empirically documented by Allayannis, Ihrig, and Weston (2001) and Kim, Mathur, and Nam (2006).

7. Conclusion

Taking foreign direct investment (FDI) as a sequential process into account, we have examined the behavior of a risk-averse multinational firm (MNF) under exchange rate uncertainty. The MNF has an investment opportunity in a foreign country. FDI is irreversible and costly expandable in that the MNF can purchase additional, but cannot sell redundant, capital at a higher unit price after the spot exchange rate has been publicly revealed. The MNF as such possesses a real (call) option that is rationally exercised whenever the foreign currency has been substantially appreciated relative to the domestic currency. The ex-post exercise of the real option convexifies the MNF's ex-ante domestic currency profit with respect to the random spot exchange rate, thereby calling for the use of currency options as a hedging instrument. We have shown that the MNF's optimal initial level of sequential FDI is always lower than that of lumpy FDI, while the expected optimal aggregate level of sequential FDI can be higher or lower than that of lumpy FDI.

If the MNF is banned from engaging in currency hedging, we have show that the MNF's ex-ante and ex-post incentives to make FDI are reduced as compared to those with perfect currency hedging. An increase in the fixed or setup cost incurred by the MNF generates similar perverse effects on FDI if the MNF's utility function satisfies the reasonable property of decreasing absolute risk aversion. Given that the change in the fixed or setup cost may be due to a change in the investment tax credit offered by the host government, or due to a change in the severity of entry barriers in the host country, FDI flows are expected to react in a predictable manner when these government policies and market conditions shift over time (Anand and Kogut, 1997; Hines, 2001). If the MNF is restricted to use the unbiased currency futures contracts as the sole hedging instrument, we have shown that the MNF's ex-ante and ex-post incentives to make FDI are enhanced as compared to those with perfect currency hedging. This implies immediately that futures hedging promotes FDI, a result consistent with the complementary nature of operational and financial hedging strategies (Allayannis, Ihrig, and Weston, 2001; Kim, Mathur, and Nam, 2006).

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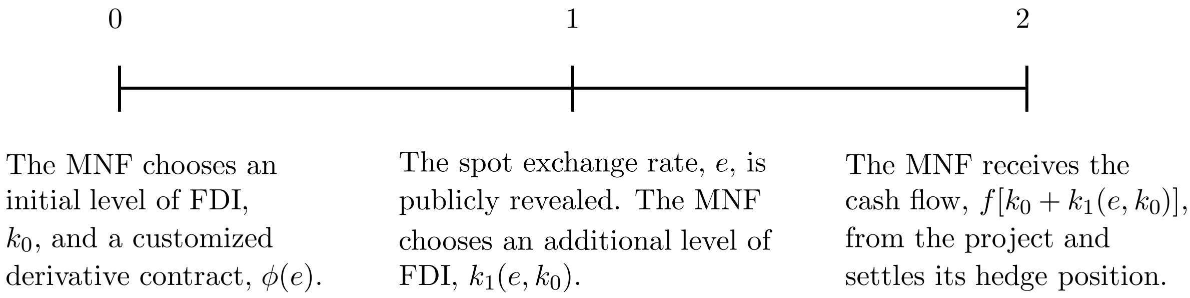

Figure 1. Time line. The underlying exchange rate uncertainty, $\tilde{e}$, is exogenously given and is resolved at $t = 1$. The MNF chooses an initial level of FDI, k_0, at the unit price of capital, p_0, and devises a customized derivative contract, $\phi(e)$, at $t = 0$. After observing the realized spot exchange rate at $t = 1$, the MNF chooses an additional level of FDI, $k_1(k_0, e)$, at the unit price of capital, $p_1 > p_0$. The MNF receives the project's cash flow, $f[k_0 + k_1(e, k_0)]$, at $t = 2$ and settles its hedge position at that time.

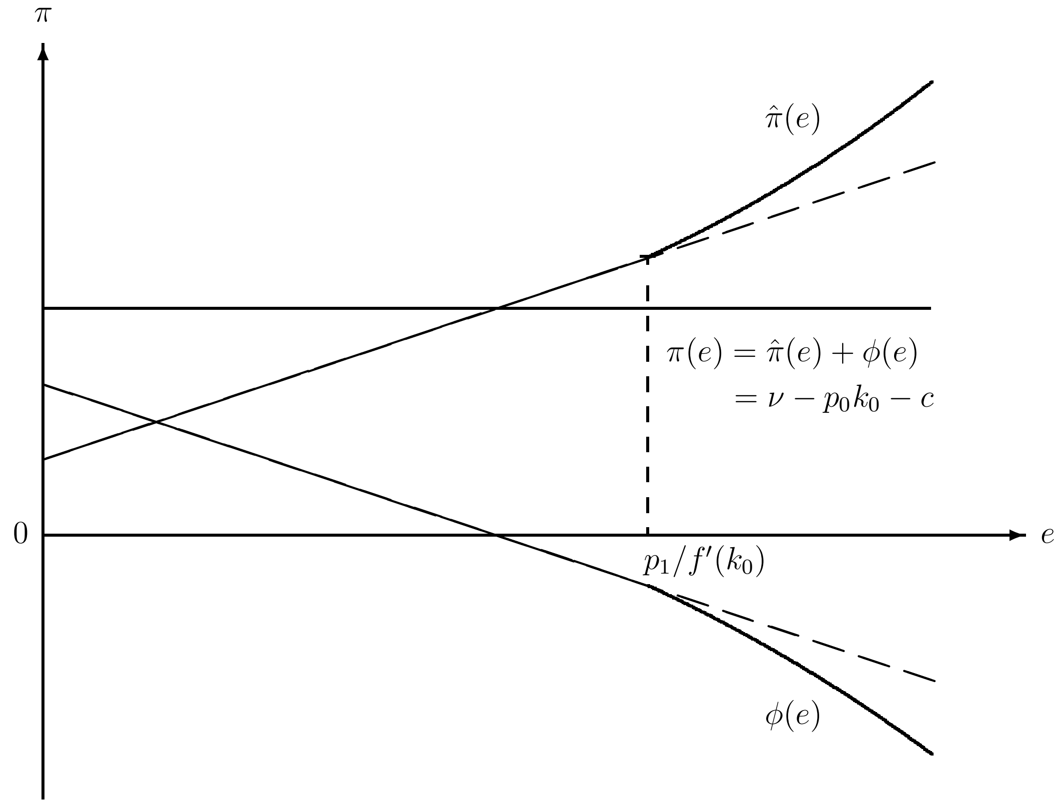

Figure 2. Hedged and unhedged domestic currency profits of the MNF. The MNF's unhedged domestic currency profit at $t = 2$ is denoted by $\hat{\pi}(e) = ef[k_0 + k_1(e, k_0)] - p_0 k_0 - p_1 k_1(e, k_0) - c$. The customized derivative contract is given by $\phi(e) = \nu - ef[k_0 + k_1(e, k_0)] + p_1 k_1(e, k_0)$, where $\nu = \mathrm{E}\{\tilde{e} f[k_0 + k_1(\tilde{e}, k_0)] - p_1 k_1(\tilde{e}, k_0)\}$. The MNF's hedged domestic currency profit at $t = 2$ is thus given by $\pi(e) = \hat{\pi}(e) + \phi(e) = \nu - p_0 k_0 - c$, which is invariant to e.